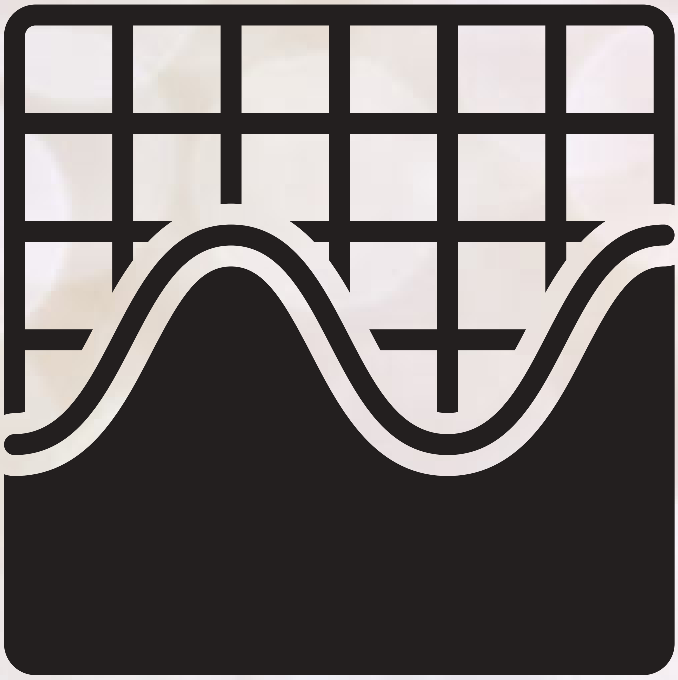

SINUSOIDAL STEADY-STATE CIRCUIT ANALYSIS

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PREFACE

In the name of ALLAH, Most Gracious, Most Merciful. All praise be to GOD S.W.T for his loving kindness and mercy, this book was successfully published.

This eBook is to help students understand, analyze circuits and calculate for Sinusoidal Steady - State Circuit Analysis. This book is compiled according to the Electrical Circuits course syllabus (DET 20033) for students of Diploma in Electronic Engineering (Computer), Diploma in Electronic Engineering (Communication) and Diploma in Electrical & Electronics Engineering under the Department of Electrical Engineering, Sultan Mizan Zainal Abidin Polytechnic, Dungun Terengganu.

The author would like to express his deepest appreciation to all parties who have given the possibility to publish this book especially friends and colleagues. Thanks are also extended to the administrative team of the Department of Electrical Engineering for their support and guidance throughout the process of preparing this ebook.

The author hopes this book can help students in their quest to succeed in this course. The author is very appreciative and invites constructive comments from readers in case of any shortcomings.

Please send your comments or suggestions to hafizah.hassan.poli@1govuc.gov .my, rusbiahty.poli@1govuc.gov.my, fadzillahhussin.poli@1govuc.gov.my. Thank you.

ABSTRACT

This Ebook is designed to assist students in performing theory task for topics Sinusoidal Steady-State Circuit Analysis. This book is organized according to the Electrical Circuits (DET20033) course syllabus for Diploma in Electronic Engineering (Computer), Diploma in Electronic Engineering (Communication) and Diploma in Electric & Electronic Engineering students in semester 2 under the Department of Electrical Engineering, Politeknik Sultan Mizan Zainal Abidin, Dungun Terengganu. This ebook contains brief notes, diagrams and sample questions along with calculation methods. It is hoped that this ebook can help students to understand more about the topic of sinusoidal steady state analysis.

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05 Apply the circuit with inductive and capacitive load

07 Apply the series and parallel R-L-C circuits.

12 Apply the combination of series-parallel R-L-C circuits

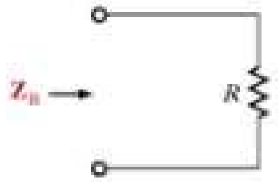
05 Understand power in AC circuits

12 Apply the understanding of the power consumption in AC circuits

Understand the AC basic circuits

Purely Resistive

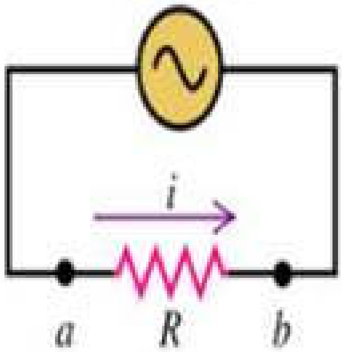
Resistor



$$Z_R = R = R\angle 0^\circ$$

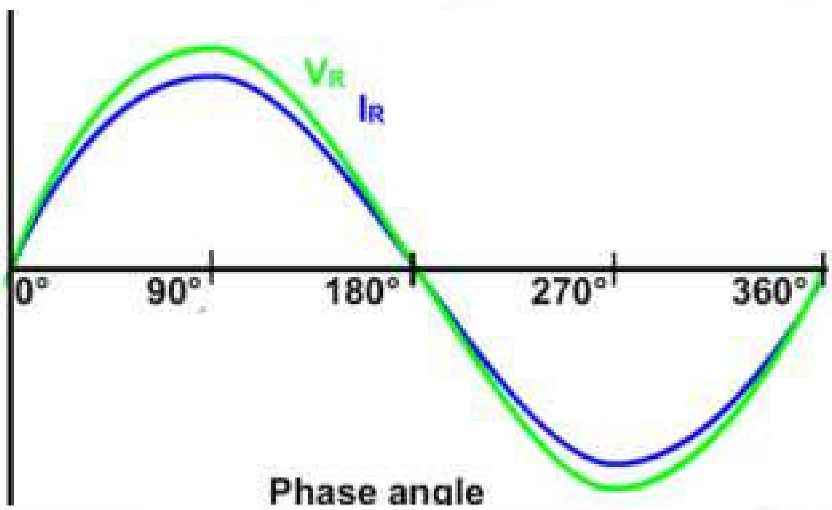
(a)

Resistor connected to ac source



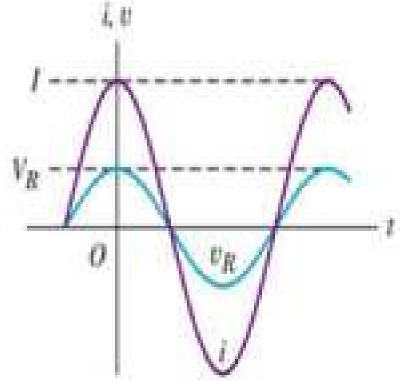
AC RESISTOR CIRCUIT

Phase relationship : There is no phase shift between V and I in a resistor

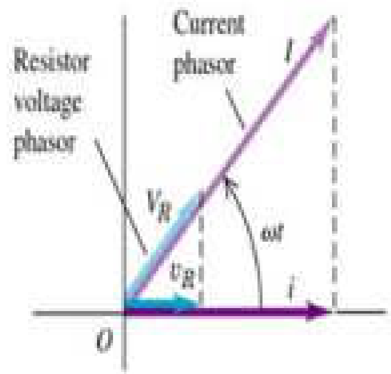


In a pure resistance, Voltage(VR) and Current (IR)
are in phase

PHASOR DIAGRAM



AC PHASE ANGLE WAVEFORM FOR RESISTIVE CIRCUIT

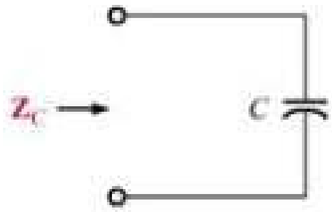


RESISTOR PHASOR (VECTOR) DIAGRAM

In a pure resistance, the phase angle between VR and IR is zero.

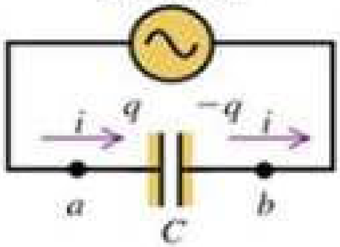
Purely Capacitive

Capacitor



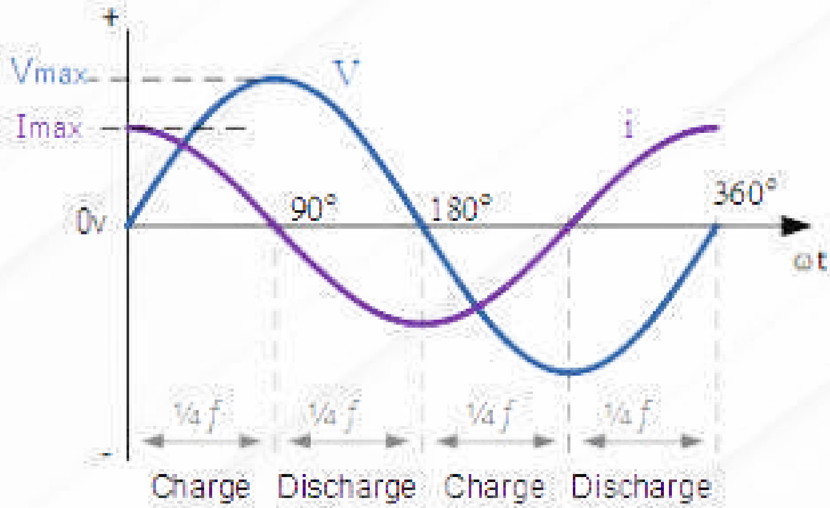
$$Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

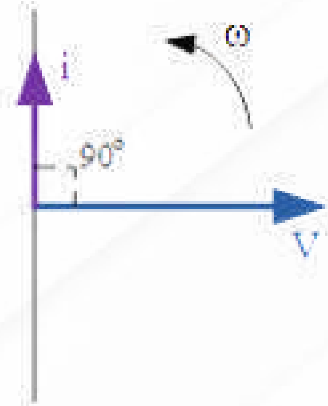
Capacitor connected to ac source



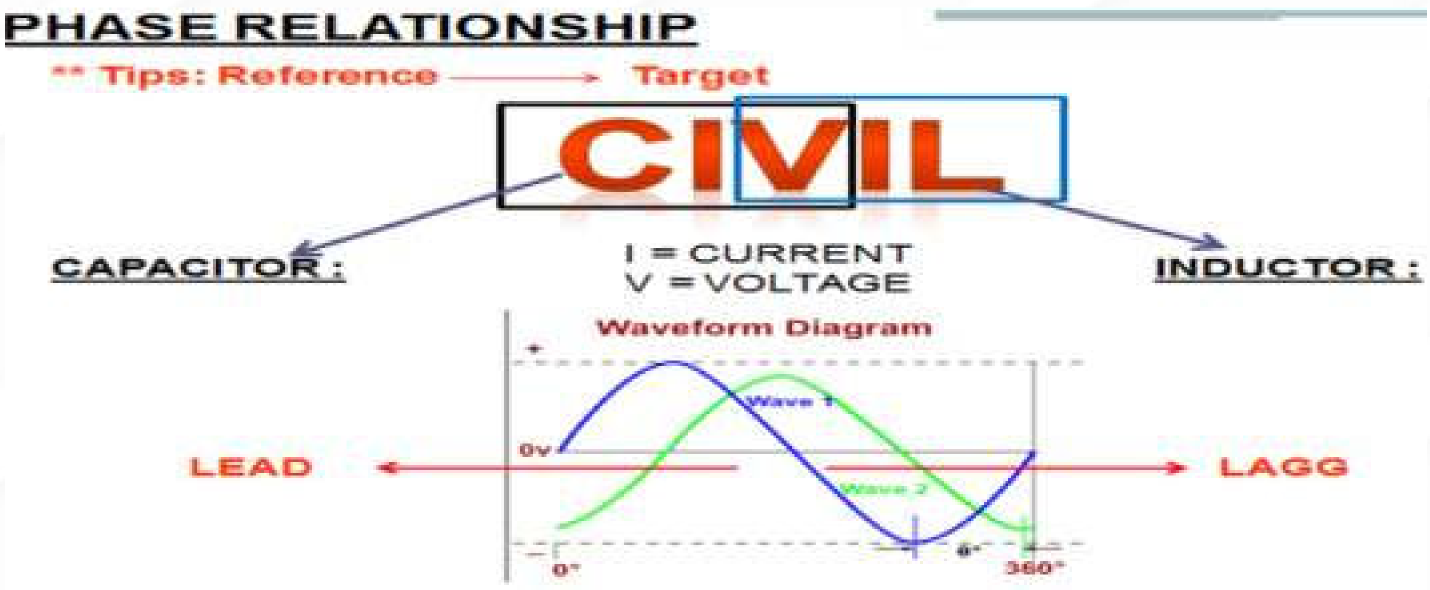
AC CAPACITOR CIRCUIT

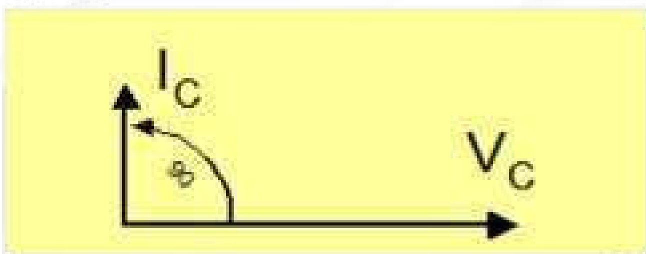
Phase relationship : The amount of phase shift between voltage and current is +90 for purely capacitive

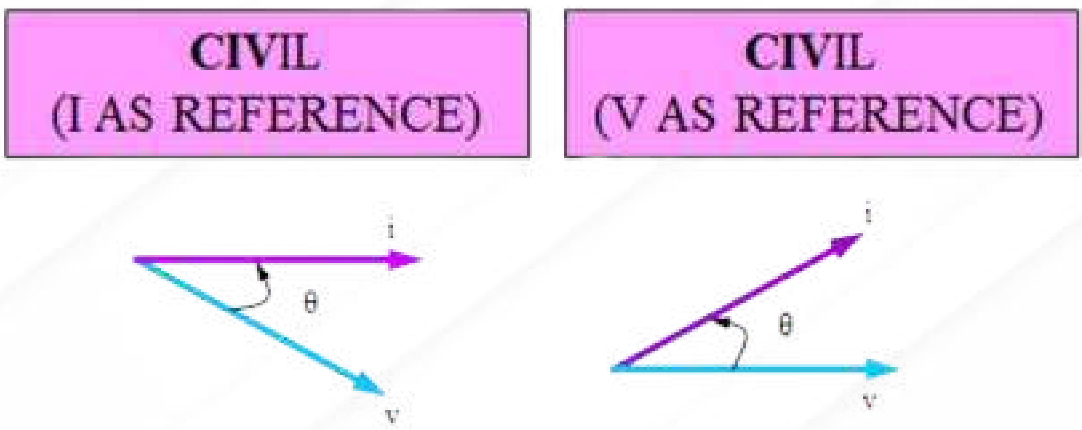




AC capacitor phasor diagram







In the capacitor, voltage LAGS charging current by 90^0
or charging current I LEADS voltage V by 90^0

CAPACITIVE REACTANCE

In resistor, the Ohm's law is V=IR, where R is the opposition to current

We will define Capacitive Reactance, XC as the opposition to the current in a capacitor.

V = I XC

XC will have units of Ohms

XC inversely proportional to f and C

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

Capacitive reactance also has phase angle associated with it

$$X_C = \frac{V}{I}$$

If V is our reference wave

$$X_C \angle \theta = \frac{V_C \angle 0^\circ}{I_C \angle +90^\circ}$$

PHASE ANGLE FOR XC

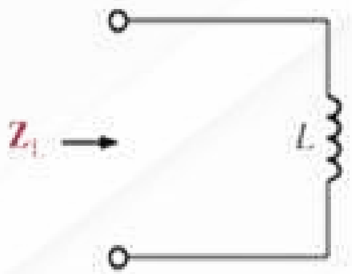
The phase angle for Capacitive Reactance (XC) will always = -90°

XC may be expressed in POLAR or RECTANGULAR form

$$X_C \angle \theta = \frac{1}{j\omega C} = 0 - jX_C = \frac{1}{\omega C} \angle -90^\circ = Z \angle -90^\circ$$

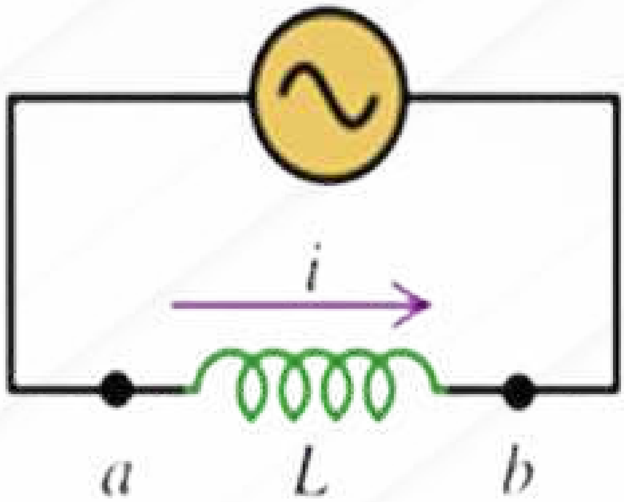
Purely Inductive

Inductor

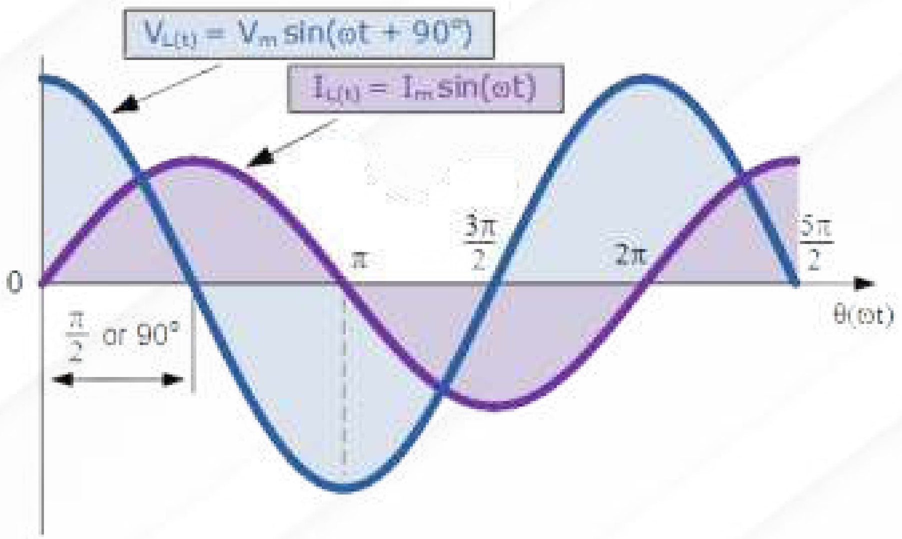


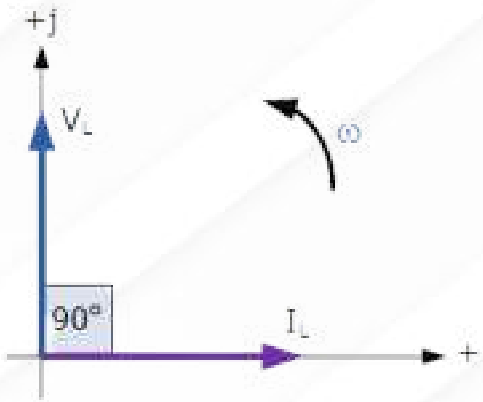
$$Z_L = j\omega L = \omega L \angle 90°$$

Inductor connected to AC source



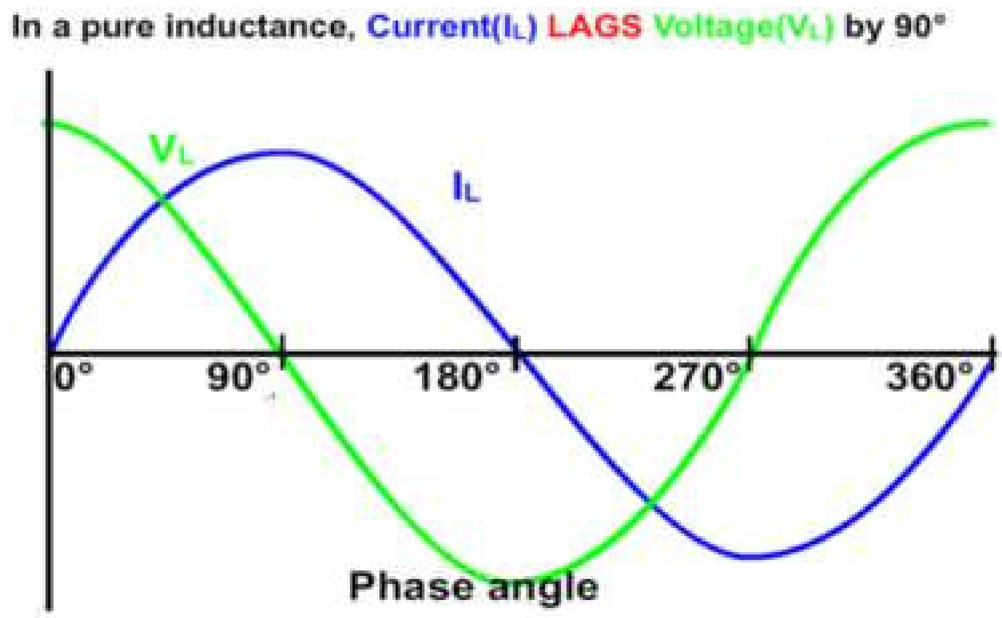
AC inductor circuit

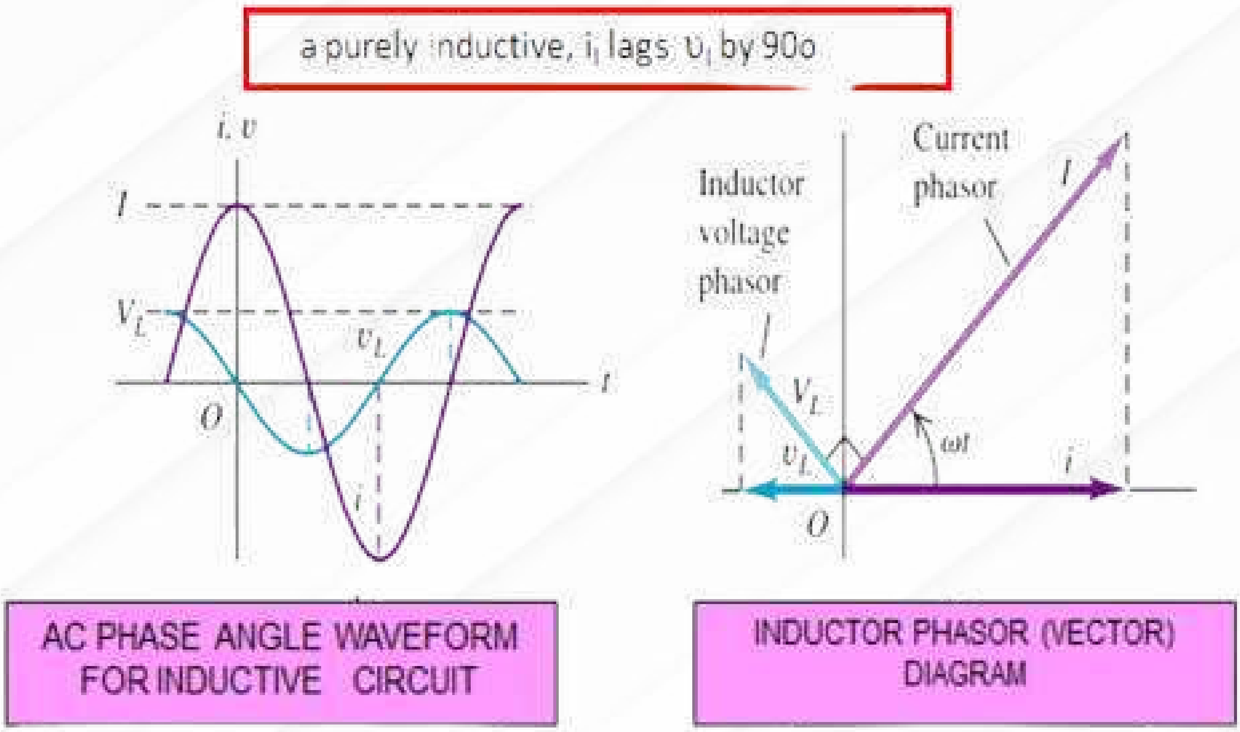




In the inductor, voltage LEADS charging current by 90 °
or charging current I LagS voltage V by 90°

CIVIL
I lags V in L





AC inductor phasor diagram

INDUCTIVE REACTANCE

Define Inductive Reactance, XL, as the opposition to current in an inductor

V = IXL

XL will have unit of ohms (Ω)

XL direct proportionality to f and L

XL = 2ϖfL = ωL

PHASE ANGLE FOR XL

If v is our reference wave

$$X_L\angle\theta = \frac{V_L\angle+90^\circ}{I_L\angle 0^\circ}$$

The phase angle for Inductive Reactance (XL) will always = 90°

XL may be expressed in POLAR or RECTANGULAR form

$$X_L\angle\theta = j\omega L = 0 + jX_L = \omega L\angle+90^\circ = Z\angle+90^\circ$$

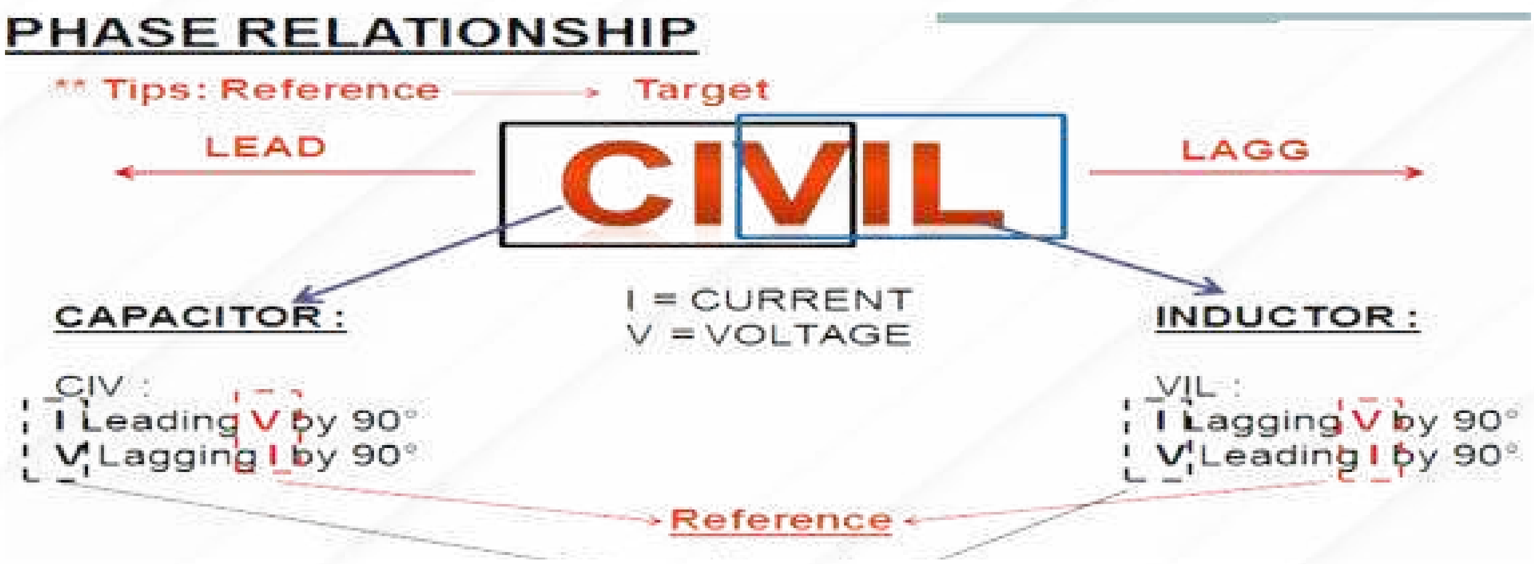
COMPARISON OF XL AND XC

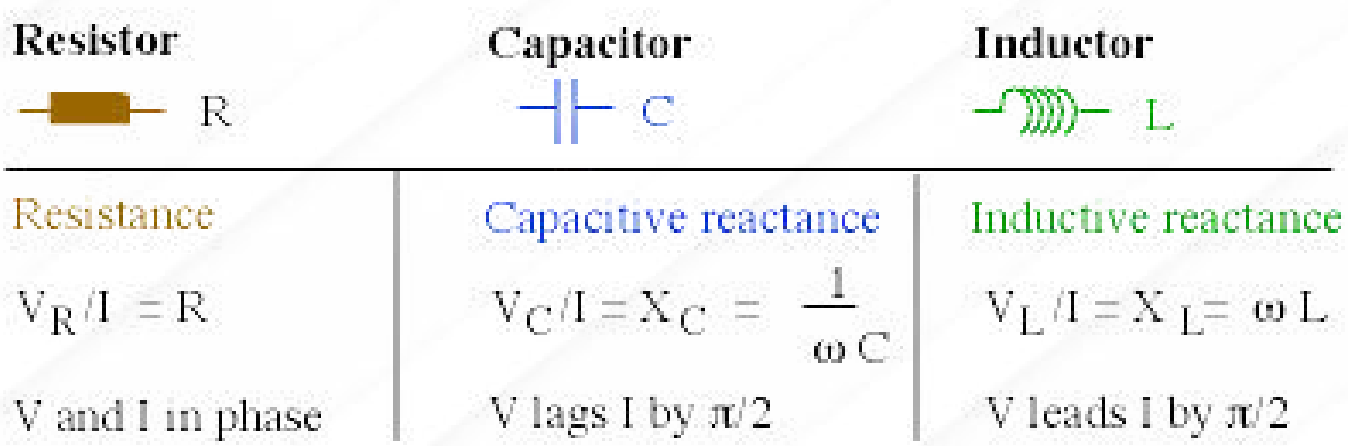
XL is directly proportional to frequency and inductance

$$XL = 2\varpi f L = \omega L$$

XC is inversely proportional to frequency and capacitance

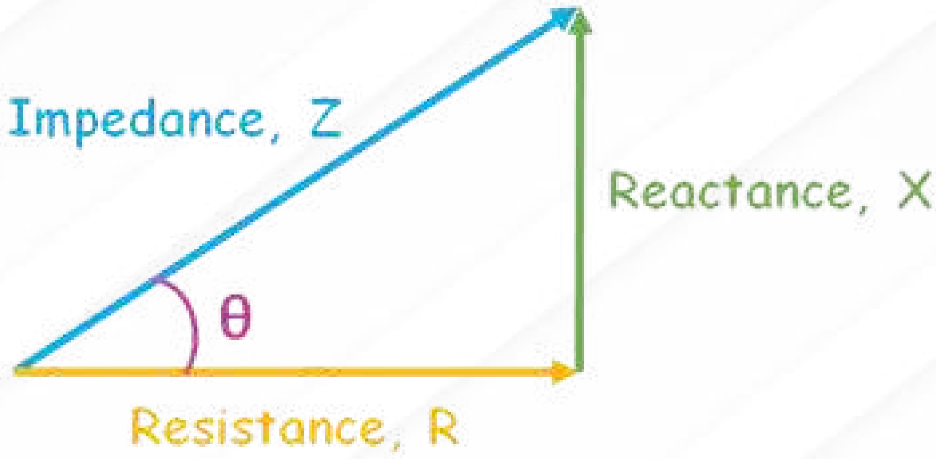
$$X_c = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$





Impedance Triangle

Impedance triangle is a geometric relationship between resistance, reactance and impedance.



The impedance triangle can be converted into a power triangle representing the three elements of power in an AC circuit.

Apply the circuit with inductive and capacitive load

Example 1:

Determine the capacitive reactance of a capacitor 10μF when connected to a circuit with input frequency 100kHz and 10kHz. Plot a graph Xc vs frequency base to the Xc value calculated above and discuss the relationship.

Solution:

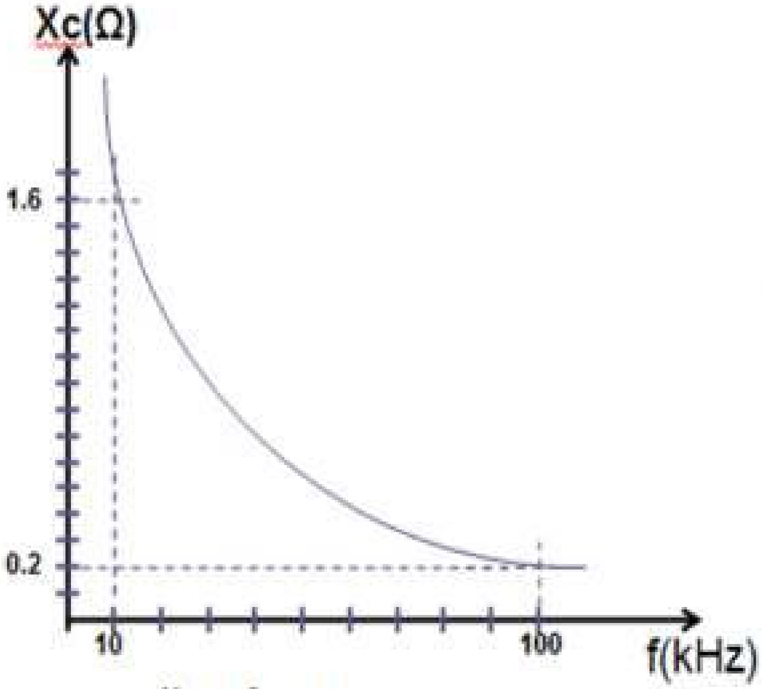
$X_c = 10\mu F \qquad X_c = \frac{1}{2\pi f C}$

$f = 100\ kHz;$

$\therefore X_c = \frac{1}{2\pi(100kHz)(10\mu F)} = 0.159\ \Omega$

$f = 10\ kHz;$

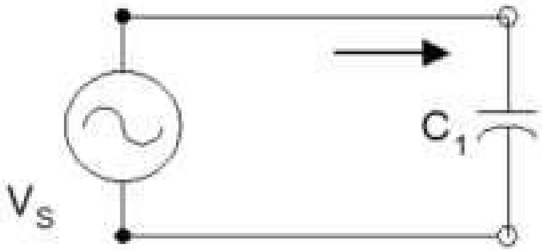
$\therefore X_c = \frac{1}{2\pi(10kHz)(10\mu F)} = 1.592\ \Omega$



Xc vs frequency

Example 2:

Ex: $f = 500$ Hz, C = 50 μF, $X_C = ?$



$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi(500\ Hz)(50\ \mu F)} = 6.366\ \Omega$$

Example 3:

Determine the inductive reactance of inductor 10mH when connected to a circuit with input frequency 100kHz and 10kHz. Plot a graph XL versus frequency base to the XL value calculated above and discuss the relationship.

Solution:

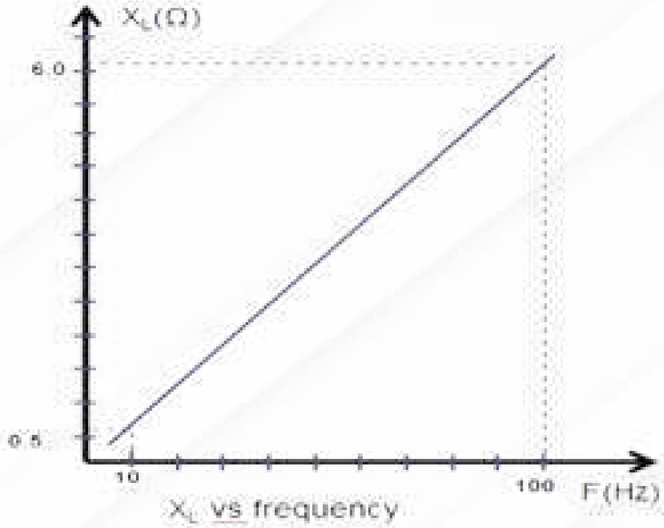
$X_L = 2\pi f L \qquad X_L = 10mH$

$f = 100\ Hz;$

$\therefore\ X_L = 2\pi(100\ Hz)(10mH) = 6.26\ \Omega$

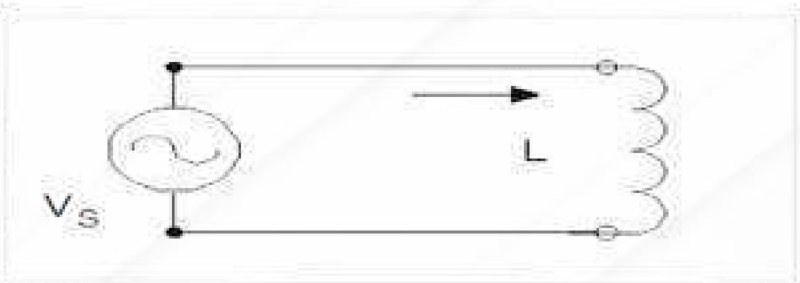
$f = 10\ Hz;$

$\therefore\ X_L = 2\pi(10Hz)(10mH) = 0.628\ \Omega$



Example 4:

- $f = 500\ \text{Hz},\ \text{L} = 500\ \text{mH},\ \text{X}_\text{L} = ?$



$$X_L = 2\pi f L = 2\pi(500Hz)(500mH) = 1.571\ k\Omega$$

Apply the series and parallel R-L-C circuits.

RC SERIES

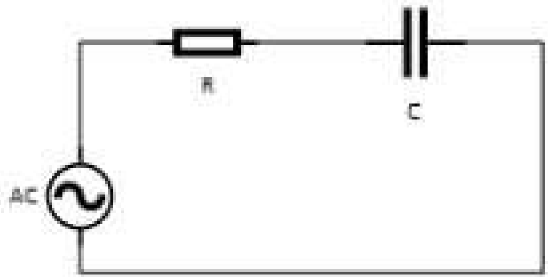


Figure 1: RC SERIES

1. $X_C = \frac{1}{2\pi f C} \Omega$
2. $Z = R - jX_C\ \Omega$
3. $I = \frac{V}{Z}$ A

Example 1

Express the total impedance and current in circuit in figure 1.

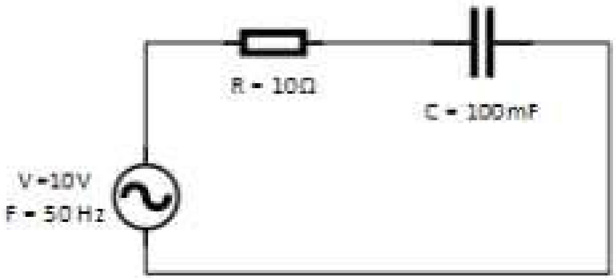


Figure 2 : RC SERIES

Solution

$$X_C = \frac{1}{2\pi(50)(100m)}$$

$= 31.83$ mΩ

Apply the combination of series-parallel R-L-C circuits

Z = 10 - j31.83m

= 10 ∠ -0.182° Ω

$$I = \frac{10\angle 0°}{10\angle -0.182°}$$

= 1 ∠ 0.182° A

RC PARALLEL

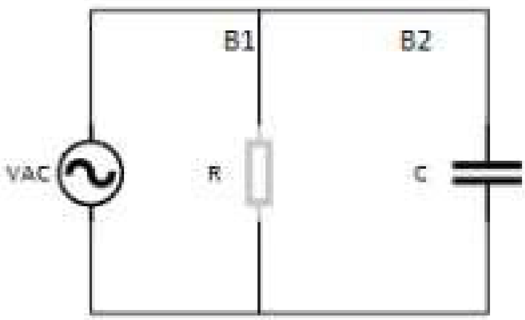


Figure 3 : RC PARALLEL

1. $X_C = \frac{1}{2\pi f C}$

METHOD 1	METHOD 2	METHOD 3
$\frac{1}{Z_T} = \frac{1}{R\angle 0°} + \frac{1}{X_C\angle -90°}$ $= \frac{R\angle 0° + X_C\angle -90°}{(R\angle 0°)(X_C\angle -90°)}$ $Z_T = \frac{(R\angle 0°)(X_C\angle -90°)}{R\angle 0° + X_C\angle -90°}\ \Omega$	Current Divider Branch 1 $I_{B1} = \frac{VAC}{R\angle 0°}$ $= I_{B1}\angle\theta°$ A Branch 2 $I_{B2} = \frac{VAC}{X_C\angle -90°}$ $= I_{B2}\angle\theta°$ A $I_{TOTAL} = I_{B1} + I_{B2}$ $= I_{B1}\angle\theta° + I_{B2}\angle\theta°$ $= I_{TOTAL}\angle\theta°$ A	Conductance, Susceptance and Admittance $Z = \frac{1}{Y\angle\theta°}$ $Y = G + jB_C$ $G = \frac{1}{R}$ $B_C = \frac{1}{X_C}$

EXAMPLE 2

For the circuit in figure 2, find the toral impedance and all currents.

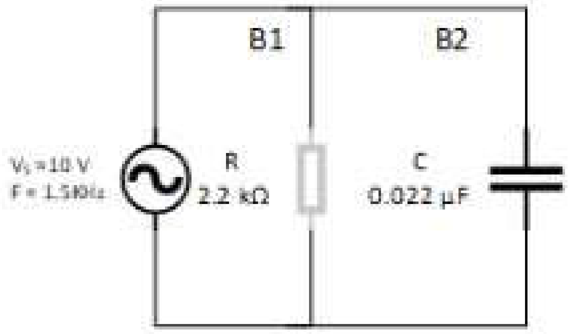


Figure 4 : RC PARALLEL

Solution

$$X_C = \frac{1}{2\pi(1.5k)(0.022\mu)}$$

$= 4.823\ k\Omega$

Total impedance	Current	METHOD 3
$\frac{1}{Z_T} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_C\angle -90^\circ}$ $\frac{1}{Z_T} = \frac{1}{2.2k\angle 0^\circ} + \frac{1}{4.823k\angle -90^\circ}$ $= \frac{2.2k\angle 0^\circ + 4.823k\angle -90^\circ}{(2.2k\angle 0^\circ)(4.823k\angle -90^\circ)}$ $Z_T = \frac{(2.2k\angle 0^\circ)(4.823k\angle -90^\circ)}{2.2k\angle 0^\circ + 4.823k\angle -90^\circ}$ $= \frac{10.61\ M\ \angle -90^\circ}{2.2\ k - j4.823k}$ $= \frac{10.61M\ \angle -90}{5.301k\ \angle -65.48^\circ}$ $= 2.002\ k\Omega\ \angle -24.52^\circ$	Branch 1 $I_{B1} = \frac{VAC}{R\angle 0^\circ}$ $= \frac{10\ \angle 0^\circ}{2.2k\ \angle 0^\circ}$ $= 4.545mA\angle\ 0^\circ$ Branch 2 $I_{B2} = \frac{VAC}{X_C\angle -90^\circ}$ $= \frac{10\ \angle 0^\circ}{4.823k\angle -90^\circ}$ $= 2.073mA\angle\ 90^\circ$ $I_{TOTAL} = I_{B1} + I_{B2}$ $= 4.545mA\angle\ 0^\circ + 2.073mA\angle\ 90^\circ$	<u>Conductance, Susceptance and Admittance</u> $G = \frac{1}{R}$ $= \frac{1}{2.2k}$ $= 454.5\ \mu S$ $B_C = \frac{1}{X_C}$ $= \frac{1}{4.823\ k}$ $= 207.3\ \mu S$ $Y = G + jB_C$ $= 454.5\ \mu S + j207.3\ \mu S$ $= 499.5\ \mu S\ \angle\ 24.52^\circ$ $Z = \frac{1}{Y\angle\theta^\circ}$

RC SERIES AND PARALLEL

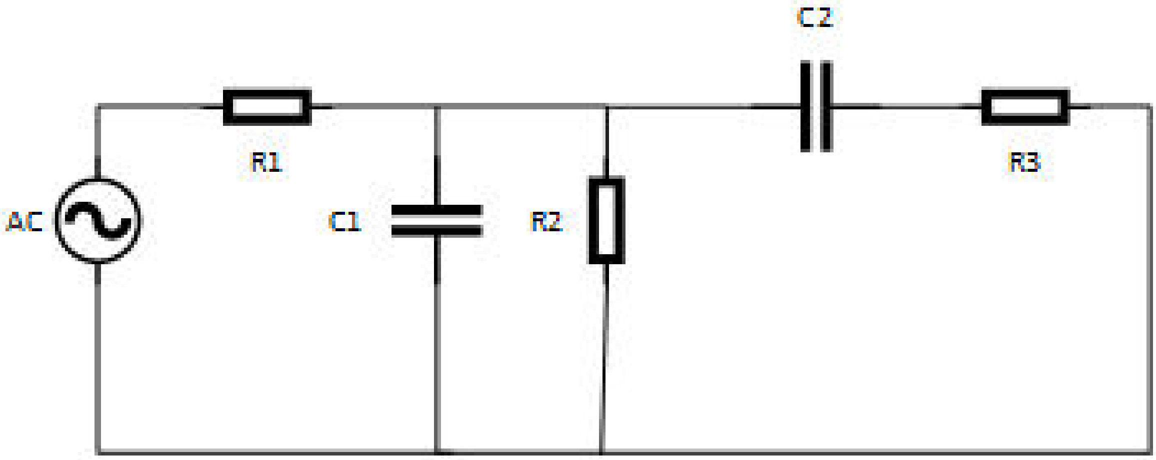


Figure 5 : RC SERIES and PARALLEL

1. $X_{C1} = \frac{1}{2\pi f C1}$

2. $X_{C2} = \frac{1}{2\pi f C2}$

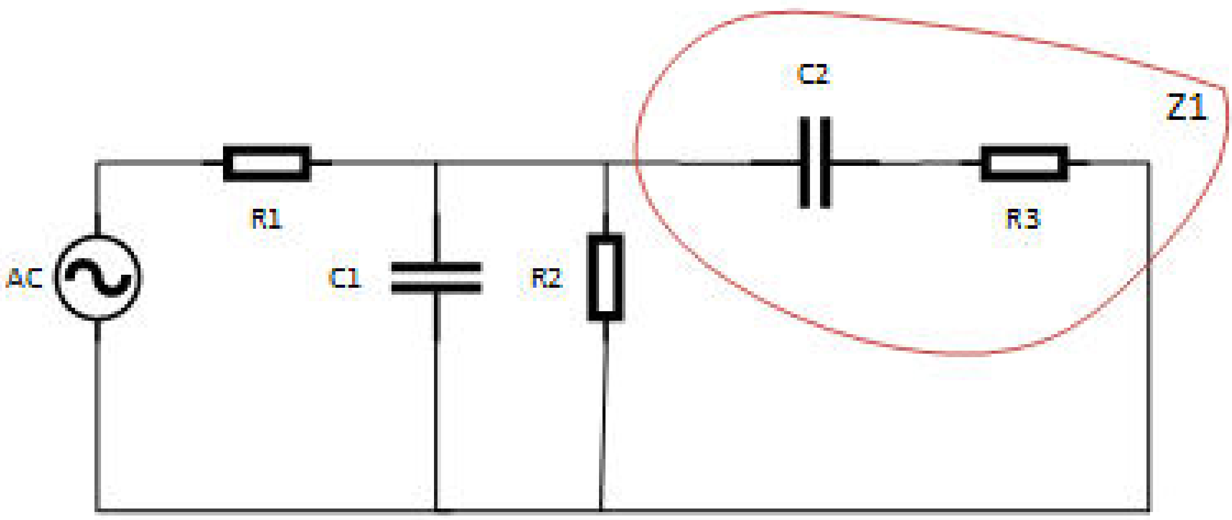


Figure 6 : RC SERIES and PARALLEL

3. $Z1 = R_3 - jX_{C2}$

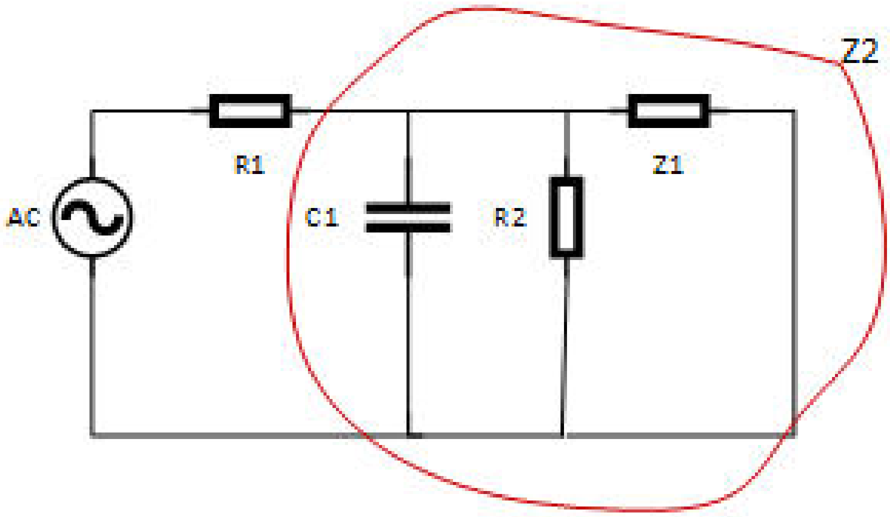


Figure 7 : RC SERIES and PARALLEL

4. $Z2 = X_{C1} \parallel R2 \parallel Z1$

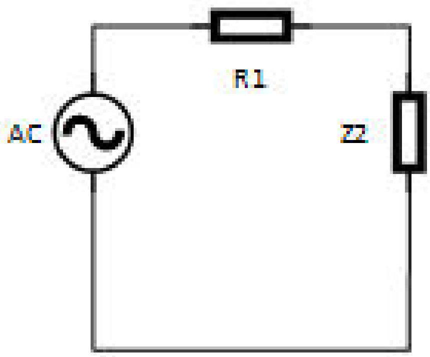


Figure 8 : RC SERIES and PARALLEL

5. $ZT = R1 + Z2$

Example 3

Express the total impedance and current in circuit in figure 9. Given VAC = 100 sin 1000t

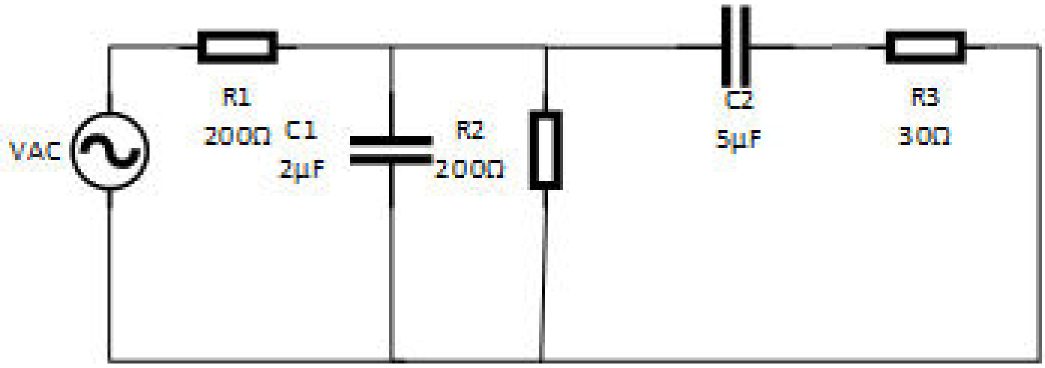


Figure 9 : RC SERIES and PARALLEL

Solution

$\omega = 1000$

$X_{C1} = \dfrac{1}{\omega C1}$

$= \dfrac{1}{1000(2\mu)}$

$= 500\ \Omega$

$X_{C2} = \dfrac{1}{\omega C2}$

$= \dfrac{1}{1000(5\mu)}$

$= 200\Omega$

$Z1 = R3 - jX_{C1}$

$= 30 - j500$

$= 500.9\ \Omega \angle -86.57°$

$Z2 = X_{C1} \parallel R2 \parallel Z1$

$$Z2 = \frac{(X_{C_1}\angle -90°)(R2\angle 0°)Z1}{R2\angle 0°(Z1)+(X_{C_1}\angle -90)Z1+(X_{C_1}\angle -90°)(R2\angle 0°)}$$

$$= \frac{(500\angle -90°)(200\angle 0°)(500.9\angle -86.57°)}{200\angle 0°(500.9\angle -86.57°)+(500\angle -90°)500.9\angle -86.57°+(500\angle -90°)(200\angle 0°)}$$

$$= \frac{50.09M\angle -176.6°}{100.2k\angle -86.57°+250.5k\angle -176.6°+100k\angle -90°}$$

$$= \frac{50.09M\angle -176.6°}{5.995k-j100k+(-250.1k-j14.86k)+(0-j100k)}$$

$$= \frac{50.09M\angle -176.6^\circ}{-244.1k - j214.9k}$$

$$= \frac{50.09M\angle -176.6^\circ}{352.2k\angle -138.6^\circ}$$

$= 142.2\ \Omega\angle\ -38^\circ$

$ZT = R1 + Z2$

$= 200\angle\ 0^\circ + 142.2\ \Omega\angle\ -38^\circ$

$= 200 + j0 + 112.1 - j87.54$

$= 312.1 - j87.54$

$= 324.1\ \Omega\ \angle\ -15.67^\circ$

$$I = \frac{100\angle 0^\circ}{324.1\ \angle -15.67^\circ}$$

$= 308.5mA\ \angle\ 15.67^\circ$

RL SERIES

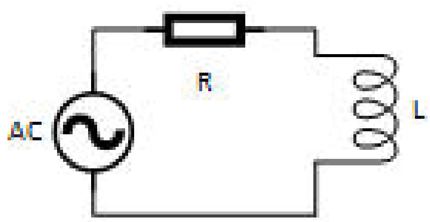


Figure 10: RL SERIES

1. $X_L = 2\pi fL\ \Omega$
2. $Z = R + jX_L\ \Omega$
3. $I = \frac{V}{Z}$ A

Example 1

Express the total impedance and current in circuit in figure 11.

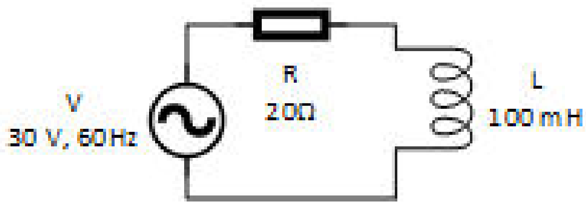


Figure 11: RL SERIES

Solution

$X_L = 2\pi fL$

$= 2\pi\,(60)\,(100m)$

$= 37.7\ \Omega$

$Z = R + jX_L$

$= 20 + j37.7$

$= 42.68\ \Omega \angle 62.05°$

$I = \frac{V}{Z}$

$= \frac{30\angle 0°}{42.68 \angle 62.05°}$

$= 0.703\ A\angle -62.05°$

RL PARALLEL

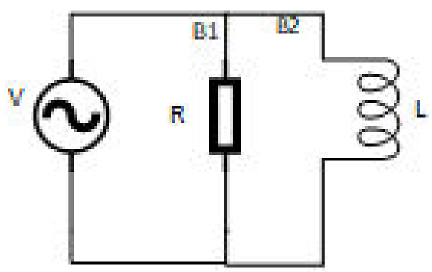


Figure 12 : RL PARALLEL

1. $X_L = 2\pi fL$

METHOD 1	METHOD 2	METHOD 3
$\frac{1}{Z_T} = \frac{1}{R\angle 0°} + \frac{1}{X_L\angle 90°}$ $= \frac{R\angle 0° + X_L\angle 90°}{(R\angle 0°)(X_L\angle 90°)}$ $Z_T = \frac{(R\angle 0°)(X_L\angle 90°)}{R\angle 0° + X_L\angle 90°}\ \Omega$	<u>Current Divider</u> Branch 1 $I_{B1} = \frac{VAC}{R\angle 0°}$ $= I_{B1}\angle\theta°$ A Branch 2 $I_{B2} = \frac{VAC}{X_L\angle 90°}$ $= I_{B2}\angle\theta°$ A $I_{TOTAL} = I_{B1} + I_{B2}$ $= I_{B1}\angle\theta° + I_{B2}\angle\theta°$ $= I_{TOTAL}\angle\theta°$ A	<u>Conductance, Susceptance and Admittance</u> $Z = \frac{1}{Y\angle\theta°}$ $Y = G - jB_L$ $G = \frac{1}{R}$ $B_L = \frac{1}{X_L}$

EXAMPLE 2

For the circuit in figure 13, find the toral impedance and all currents.

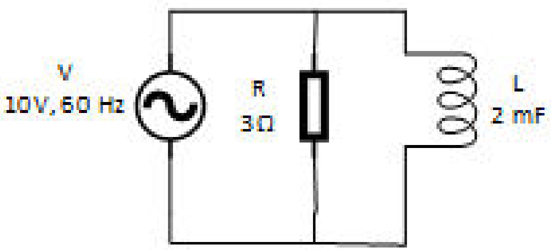


Figure 13 : RL PARALLEL

Solution

1. $X_L = 2\pi fL$
$= 2\pi\,(60)\,(2m)$
$= 0.754\ \Omega$

Total impedance	Current	METHOD 3
$\frac{1}{Z_T} = \frac{1}{R\angle 0°} + \frac{1}{X_L\angle 90°}$	Branch 1	<u>Conductance, Susceptance and Admittance</u>

$\frac{1}{Z_T} = \frac{1}{3\angle 0^\circ} + \frac{1}{0.754\angle 90^\circ}$ $= \frac{3\angle 0^\circ + 0.754\angle 90^\circ}{(3\angle 0^\circ)(0.754\angle 90^\circ)}$ $Z_T = \frac{(3\angle 0^\circ)(0.754\angle 90^\circ)}{3\angle 0^\circ + 0.754\angle 90^\circ}$ $= \frac{2.262\angle 90^\circ}{3+j0.754}$ $= \frac{2.262\angle 90^\circ}{3.093\angle 14.11^\circ}$ $= 0.731\Omega\angle 75.89^\circ$	$I_{B1} = \frac{VAC}{R\angle 0^\circ}$ $= \frac{10\angle 0^\circ}{3\angle 0^\circ}$ $= 3.333\ A\angle 0^\circ$ Branch 2 $I_{B2} = \frac{VAC}{X_L\angle 90^\circ}$ $= \frac{10\angle 0^\circ}{0.754\angle 90^\circ}$ $= 13.26\ A\angle -90^\circ$ $I_{TOTAL} = I_{B1} + I_{B2}$ $= 3.333A\angle 0^\circ + 13.26A\angle -90^\circ$ $= 3.333 + j0 + 0 - j13.26$ $= 13.67A\angle -75.89^\circ$	$G = \frac{1}{R}$ $= \frac{1}{3}$ $= 333.3\ mS$ $B_L = \frac{1}{X_L}$ $= \frac{1}{0.754}$ $= 1.326S$ $Y = G - jB_L$ $= 333.3\ mS - j1.326S$ $= 1.367S\angle -75.89^\circ$ $Z = \frac{1}{Y\angle\theta^\circ}$ $= \frac{1}{1.367\angle -75.89^\circ}$ $= 0.732\Omega\angle 75.89^\circ$

RL SERIES AND PARALLEL

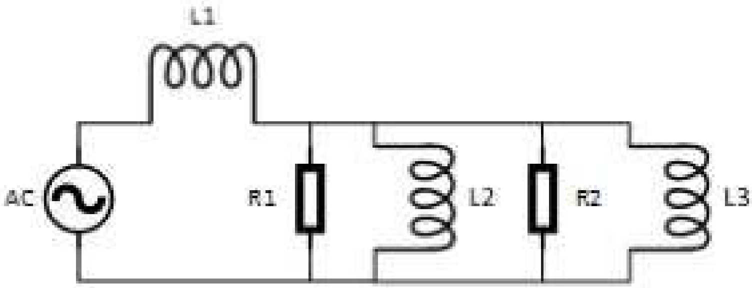


Figure 14: RL SERIES and PARALLEL

1. $X_{L1} = 2\pi fL1$
2. $X_{L2} = 2\pi fL2$
3. $X_{L3} = 2\pi fL3$

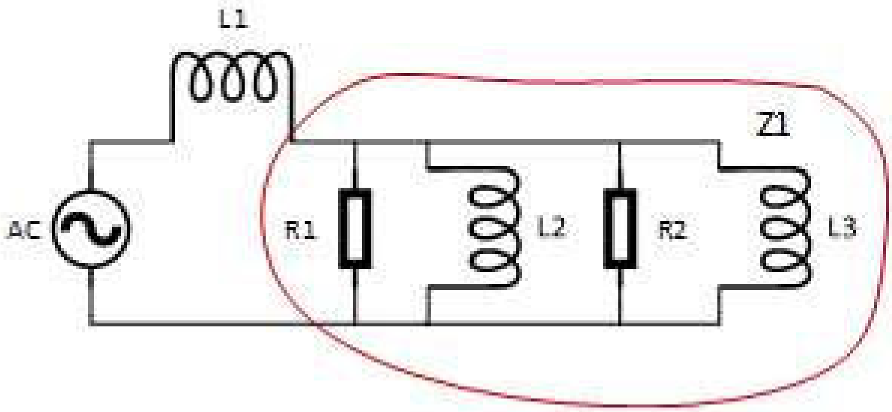


Figure 15 : RL SERIES and PARALLEL

4. $Z_1 = R_1 \parallel X_{L2} \parallel R_2 \parallel X_{L3}$

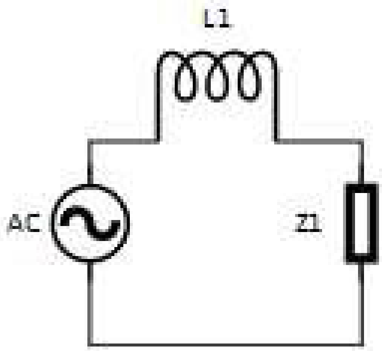


Figure 16 : RL SERIES and PARALLEL

5. $ZT = Z1 + jXL_1$

EXAMPLE 3

Express the total impedance and current in circuit in figure 17.

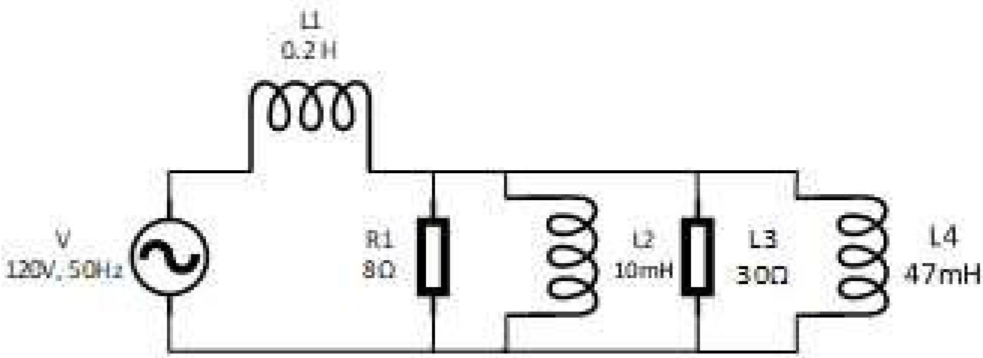


Figure 17 : RL SERIES and PARALLEL

Solution

$X_{L1} = 2\pi f L1$
$= 2\pi\ (50)\ (0.2)$
$= 62.83\ \Omega$

$X_{L2} = 2\pi f L2$
$= 2\pi\ (50)\ (10m)$
$= 3.142\ \Omega$

$X_{L3} = 2\pi f L3$
$= 2\pi\ (50)\ (47m)$
$= 14.77\ \Omega$

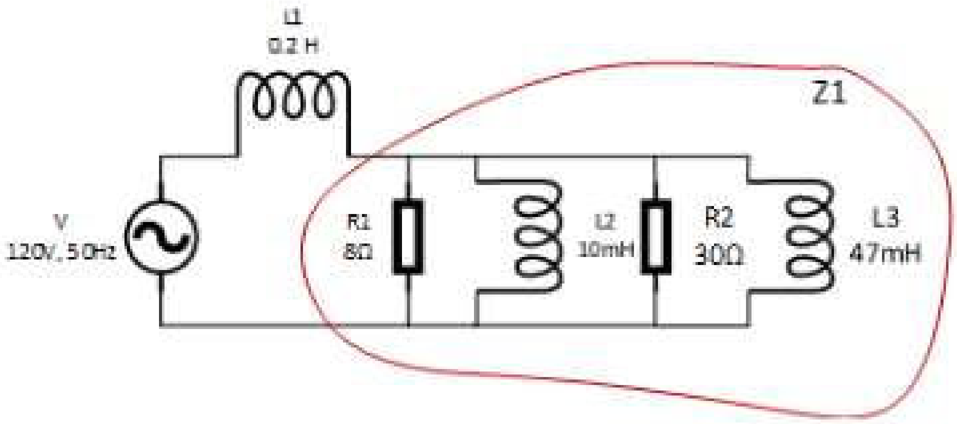


Figure 18 : RL SERIES and PARALLEL

$Z_1 = R_1 \parallel X_{L2} \parallel R_2 \parallel X_{L3}$

$$= \frac{R_1\angle 0^\circ (X_{L2}\angle 90^\circ)(R_2\angle 0^\circ)(X_{L3}\angle 90^\circ)}{(X_{L2}\angle 90^\circ)(R_2\angle 0^\circ)(X_{L3}\angle 90^\circ) + R_1\angle 0^\circ(R_2\angle 0^\circ)(X_{L3}\angle 90^\circ) + R_1\angle 0^\circ(X_{L2}\angle 90^\circ)(X_{L3}\angle 90^\circ) + (X_{L2}\angle 90^\circ)(R_2\angle 0^\circ)(X_{L3}\angle 90^\circ)}$$

$$= \frac{8\angle 0^\circ (3.142\angle 90^\circ)(30\angle 0^\circ)(14.77\angle 90^\circ)}{(3.142\angle 90^\circ)(30\angle 0^\circ)(14.77\angle 90^\circ) + 8\angle 0^\circ(30\angle 0^\circ)(14.77\angle 90^\circ) + 8\angle 0^\circ(3.142\angle 90^\circ)(14.77\angle 90^\circ) + 8\angle 0^\circ(3.142\angle 90^\circ)(30\angle 0^\circ)}$$

$$= \frac{11.14k\angle 180^\circ}{(1.392k\angle 180) + 3.545K\angle 90^\circ + 371.3\angle 180^\circ + 754.1\angle 90^\circ}$$

$$= \frac{11.14k\angle 180^\circ}{(-1.392k + j0) + j3.545k + (-371.3 + j0) + j754.1}$$

$$= \frac{11.14k\angle 180^\circ}{-1.763k + j4.2991k}$$

$$= \frac{11.14k\angle 180^\circ}{4.647k\angle 112.3^\circ}$$

$= 2.397\Omega\angle 67.7^\circ$

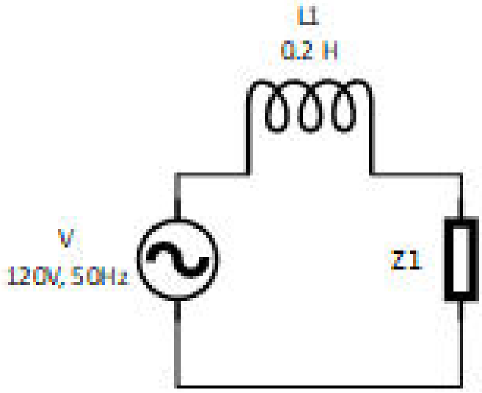


Figure 19 : RL SERIES and PARALLEL

$ZT = Z1 + jXL_1$

$= 2.397\angle 67.7° + j62.83$

$= 0.91 + j\,2.218 + j62.83$

$= 0.91 + j\,65.05$

$= 65.05\Omega\angle 89.2°$

$I = \frac{V}{Z}$

$= \frac{120\angle 0°}{65.05\angle 89.2°}$

$= 1.845\ A\ \angle -89.2°$

RLC SERIES

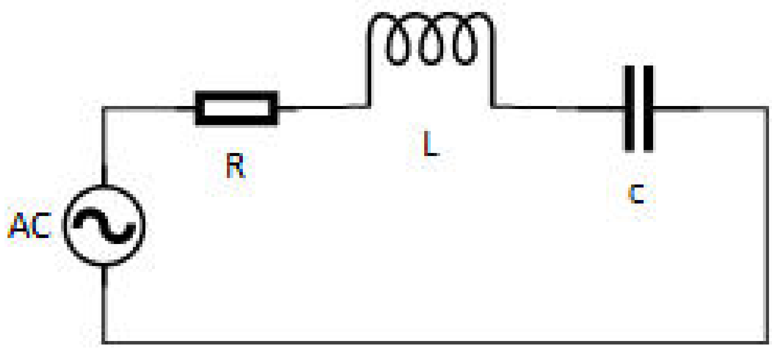


Figure 20: RLC SERIES

1. $X_L = 2\pi fL\ \Omega$

2. $X_C = \frac{1}{2\pi fC}\ \Omega$

2. $Z = R + jX_L - jX_C\ \Omega$

3. $I = \frac{V}{Z}$ A

Example 1

Express the total impedance in circuit in figure 21.

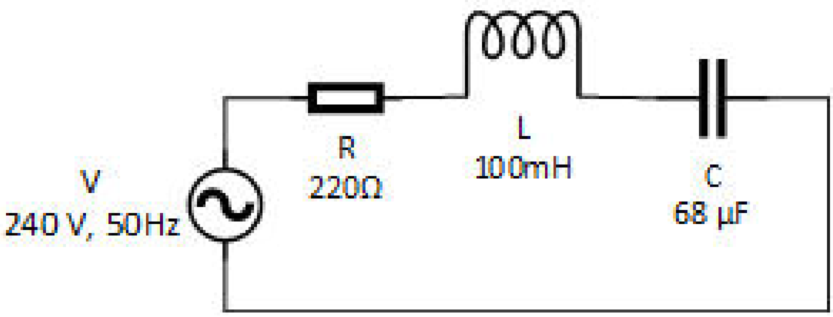


Figure 21 : RLC SERIES

Solution

$X_L = 2\pi fL$

$= 2\pi\ (50)\ (100m)$

$= 31.42\ \Omega$

$X_C = \frac{1}{2\pi fC}$

$= \frac{1}{2\pi(50)(68\mu)}$

$= 46.81\ \Omega$

$Z = R + jX_L - jX_C$

$= 220 + j31.42 - j46.81$

$= 220.5 \angle -4°\ \Omega$

$I = \frac{V}{Z}$

$= \frac{240\angle 0°}{220.5\angle -4°}$

$= 1.088\ A\angle 4°$

RLC PARALLEL

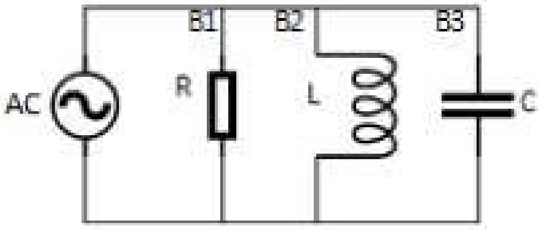


Figure 22 : RLC PARALLEL

1. $X_L = 2\pi fL\ \Omega$
2. $X_C = \frac{1}{2\pi fC}\ \Omega$

<table>
<tr><th>METHOD 1</th><th>METHOD 2</th></tr>
<tr><td>$\frac{1}{Z_T} = \frac{1}{R\angle 0°} + \frac{1}{X_L\angle 90°} + \frac{1}{X_C\angle -90°}$

$= \frac{R\angle 0°(X_C\angle -90°) + (X_C\angle -90°)(X_L\angle 90°) + R\angle 0°(X_L\angle 90°)}{(R\angle 0°)(X_L\angle 90°)(X_C\angle -90°)}$

$Z_T = \frac{(R\angle 0°)(X_L\angle 90°)(X_C\angle -90°)}{R\angle 0°(X_C\angle -90°) + (X_C\angle -90°)(X_L\angle 90°) + R\angle 0°(X_L\angle 90°)}\ \Omega$</td><td><u>Current Divider</u>

Branch 1

$I_{B1} = \frac{VAC}{R\angle 0°}$
$= I_{B1}\angle\theta°\ A$

Branch 2

$I_{B2} = \frac{VAC}{X_L\angle 90°}$
$= I_{B2}\angle\theta°\ A$

Branch 3

$I_{B3} = \frac{VAC}{X_C\angle -90°}$
$= I_{B3}\angle\theta°\ A$

$I_{TOTAL} = I_{B1} + I_{B2} + I_{B3}$
$= I_{B1}\angle\theta° + I_{B2}\angle\theta° + I_{B3}\angle\theta°$
$= I_{TOTAL}\angle\theta°\ A$</td></tr>
</table>

METHOD 3
Conductance, Susceptance and Admittance $Z = \frac{1}{Y \angle \theta°}$ $Y = G - jB_L + jB_C$ $G = \frac{1}{R}$ $B_L = \frac{1}{X_L}$ $B_C = \frac{1}{X_C}$

EXAMPLE 2

For the circuit in figure 23, find the toral impedance and all currents.

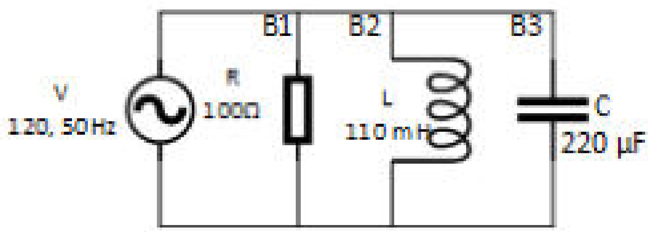


Figure 23: RLC PARALLEL

$X_L = 2\pi fL$
$= 2\pi\,(50)\,(110m)$
$= 34.56\ \Omega$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi(50)(220\mu)}$$

$$= 14.46\ \Omega$$

METHOD 1

$$\frac{1}{Z_T} = \frac{1}{R\angle 0°} + \frac{1}{X_L\angle 90°} + \frac{1}{X_C\angle -90°}$$

$$= \frac{1}{100\angle 0°} + \frac{1}{34.56\angle 90°} + \frac{1}{14.46\angle -90°}$$

$$= \frac{100\angle 0°(14.46\angle -90°)+(14.46\angle -90°)(34.56\angle 90°)+100\angle 0°(34.56\angle 90°)}{(100\angle 0°)(34.56\angle 90°)(14.46\angle -90°)}$$

$$Z_T = \frac{(100\angle 0°)(34.56\angle 90°)(14.46\angle -90°)}{100\angle 0°(14.46\angle -90°)+(14.46\angle -90°)(34.56\angle 90°)+100\angle 0°(34.56\angle 90°)}$$

$$= \frac{49.97k\angle 0°}{1.446k\angle -90°+(499.7\angle 0°)+100\angle 0°(3.456k\angle 90°)}$$

$$= \frac{49.97k\angle 0°}{-j1.446k+499.7+100+j3.456k}$$

$$= \frac{49.97k\angle 0°}{599.7+j2.01k}$$

$$= \frac{49.97k\angle 0°}{2.098k\angle 73.39°}$$

$= 23.82 \angle -73.39° \ \Omega$

METHOD 2

<u>Current Divider</u>

Branch 1

$$I_{B1} = \frac{VAC}{R\angle 0°}$$

$$= \frac{120\angle 0°}{100\angle 0°}$$

$= 1.2 \text{ A} \angle 0°$

Branch 2

$$I_{B2} = \frac{VAC}{X_L\angle 90°}$$

$$= \frac{120\angle 0°}{34.56\angle 90°}$$

$= 3.472\angle -90° \text{ A}$

Branch 3

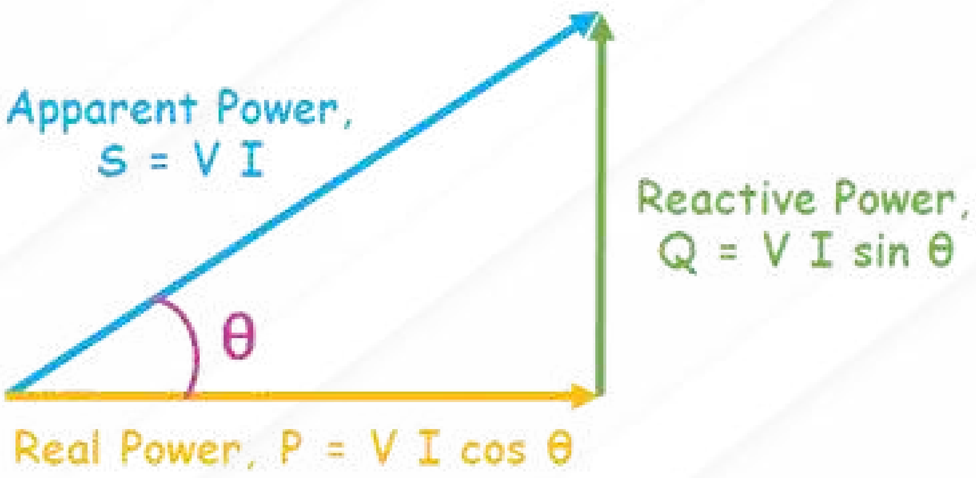
Understand power in AC circuits

In an AC circuit, the instantaneous values of the voltage, current and therefore power are constantly changing being influenced by the supply. So the power in AC circuits can not be calculated in the same in DC circuits, but power (p) is still equal to the voltage (v) times the amperes (i).
AC circuits also contain reactance, so there is a power component as a result of the magnetic or electric fields created by the components.

The result is that, unlike a purely resistive component, this power is stored and then returned back to the supply as the sinusoidal waveform goes through one complete periodic cycle.
Thus, the average power absorbed by a circuit is the sum of the power stored and the power returned over one complete cycle.

Power Triangle

Power Triangle is a right-angled triangle whose sides represent the real, reactive and apparent power. Base, perpendicular and hypotenuse of this right-angled triangle denotes the Real, Reactive and Apparent power respectively



Apparent Power

- Apparent power is the power supplied
- The combination of reactive power and true power
- It is the product of a circuit's voltage and current, without reference to phase voltage
- It is a function of a circuit's total impedance, Z.
- Symbol : S
- Unit : VA (Volt-Ampere)
- **S = V I**

Reactive Power

- Sometimes called wattless power
- Reactive power is the power that is stored (either the inductor or capacitor) and returned to the source of supply
- It is caused by the existence of the components of strain (XL or XC)
- Symbol : Q
- Unit : VAR (Volt-Amps-Reactive)
- **Q = V I sin θ**

Real Power

- Also known as true or active power
- Real power is the amount of power used or dissipated in AC circuits.
- It is a function of a circuit's dissipative elements, usually resistance, R
- Symbol : P
- Unit : W (Watt)
- $\mathbf{P = V\,I\,\cos\theta}$

Power Factor

- The power factor is the ratio between true power and apparent power
- It is unitless quantity and generally expressed as either a decimal value, for example 0.95, or as a percentage: 95%.
- It is often desirable to adjust the power factor of a system to near 1.0
- A high power factor is generally desirable in a transmission system to reduce transmission losses and improve voltage regulation at the load.

$$pf = \frac{Real\ Power}{Apparent\ Power} = \frac{V\,I\cos\theta}{V\,I} = \cos\theta$$

- Where "θ" is the difference between the phase angles of the voltage and the current , $\theta=\theta v-\theta i$.
- The power factor varies between 0 and 1, depending on the load.

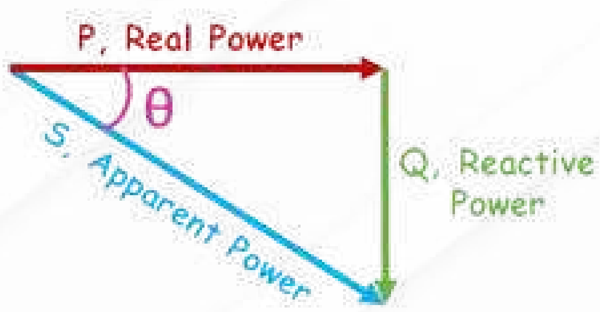
Power Factor

Leading Power Factor

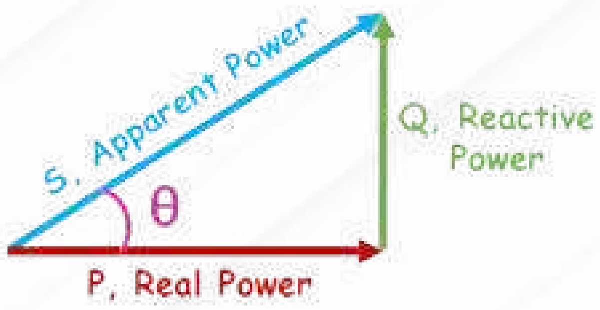
- Power factor is described as leading when the current waveform is advanced in phase with respect to voltage.
- A leading power factor signifies that the load is capacitive, as the load “supplies” reactive power.
- The reactive component Q is negative as reactive power is being supplied to the circuit.

Lagging Power Factor

- Power factor is described as lagging when the current waveform is behind the voltage waveform
- A lagging power factor signifies that the load is inductive, as the load will “consume” reactive power.
- The reactive component Q is positive as reactive power travels through the circuit and is “consumed” by the inductive load.



Leading Power Factor



Lagging Power Factor

Power Factor

For Purely Resistive Load

- The current and voltages waveforms are in phase with each other so the phase difference is $0°$
- Thus the power factor is

$$pf = \cos\theta = \cos 0° = 1$$

- Where pf=1 indicates that the maximum power is delivered.
- The number of watts consumed is the same as the number of volt-amperes consumed
- It is referred to a unity power factor.

For Purely Reactive Load

- The current and voltage waveforms are out-of-phase with each other by $90°$ so the phase difference between v and i is $90°$
- Thus the power factor is

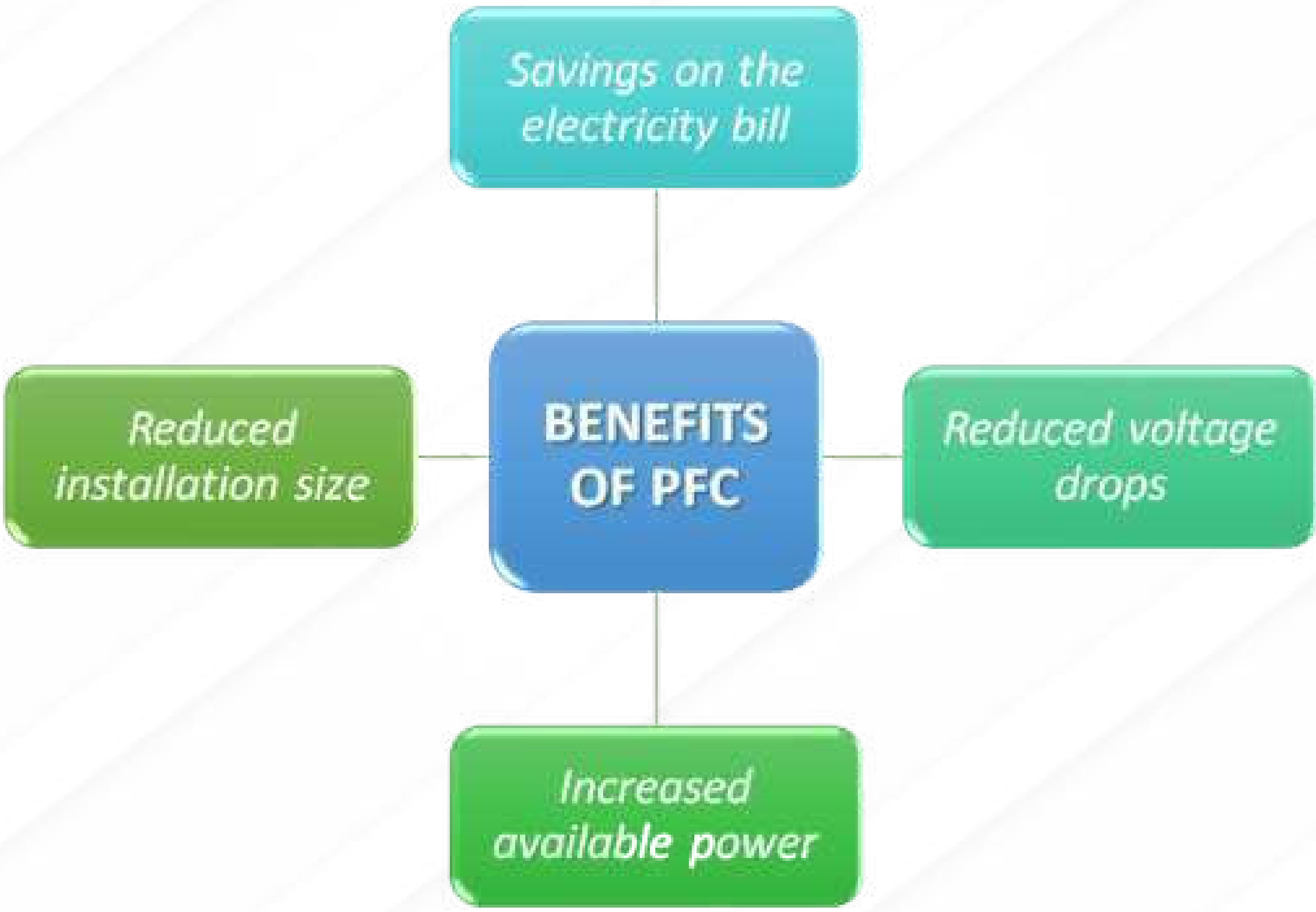
$$pf = \cos\theta = \cos 90° = 0$$

Power Factor Correction

- Power factor correction (PFC) aims to improve power factor, and therefore power quality

- PFC tries to push the power factor of the electrical system such as the power supply towards 1, and even though it doesn't reach this it gets to as close as 0.95 which is acceptable for most applications.

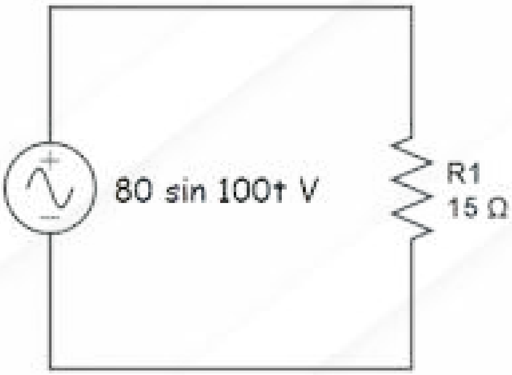
- By supplying or absorbing reactive power, adding capacitors or inductors that act to cancel the inductive or capacitive effects of the load, respectively.



Apply the understanding of the power consumption in AC circuits

Example 1

For the resistive circuit shown, find the real power absorbed by the resistor.



Solution :

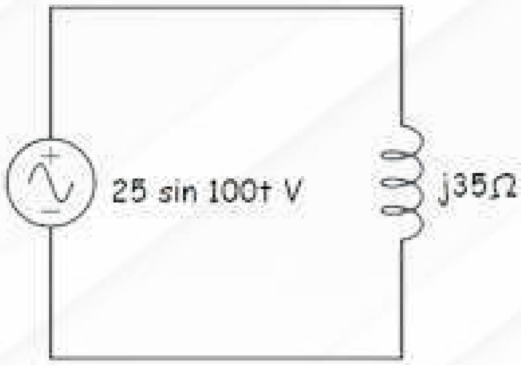
$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{80}{\sqrt{2}} = 56.57V$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{56.57}{15} = 3.77\ A$$

$$P = V_{rms} I_{rms} = (56.57)(3.77) = 213.27W$$

Example 2

For the inductive circuit shown, find the reactive power of the inductor.



Solution :

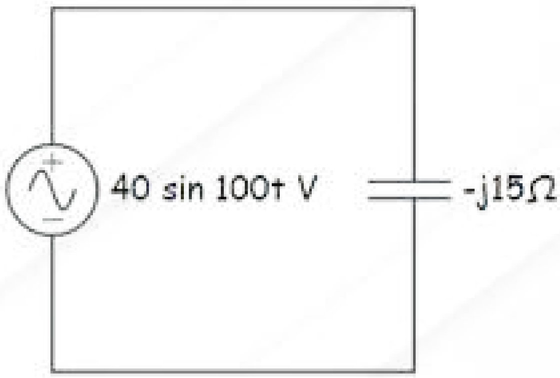
$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{25}{\sqrt{2}} = 17.68V$$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{17.68}{j35} = 0.505 < -90^o\ A$$

Q= $V_{rms} I_{rms} sin\theta$ = (17.68)(0.505)sin(90^o) = 8.928VAR

Example 3

For the capacitive circuit shown below, find the reactive power of the capacitor.



Solution :

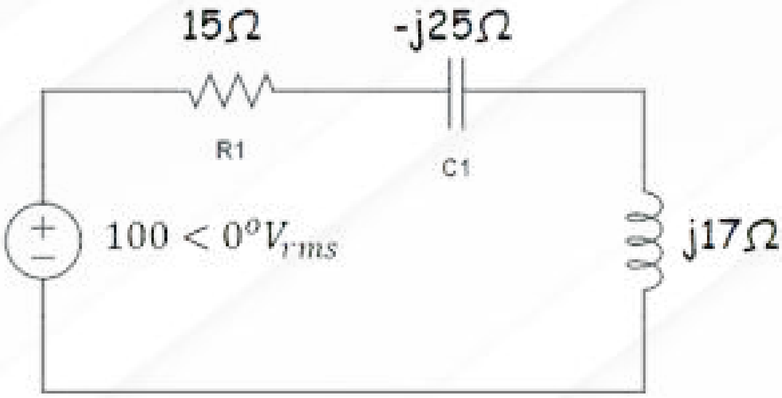
$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28V$$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{28.28}{-j15} = 1.885 < 90^o\ A$$

Q= $V_{rms} I_{rms} sin\theta$ = (28.28)(1.885)sin(90^o) = 53.31VAR

Example 4

Determine the real, reactive and apparent power of the circuit below.



Solution :

Total impedance,

$$Z_T = R + jX_L - jX_C = 15 + 17j - 25j$$
$$Z_T = 15 - 8j\Omega = 17 < -28.07^o\Omega$$

Total current flow through the circuit,

$$I_{rms} = \frac{V_{rms}}{Z_T} = \frac{100 < 0^o}{17 < -28.07^o} = 5.88 < -28.07^o\text{A}$$

Real power, P

$$P = V_{rms}I_{rms}\cos\theta = (100)(5.88)\cos 28.07^o = 518.84^o W$$

Reactive power, Q

$$Q = V_{rms}I_{rms}\sin\theta = (100)(5.88)\sin 28.07^o = 276.68^o VAR$$

Apparent power, S

$$S = V_{rms}I_{rms} = (100)(5.88) = 588VA$$

Example 5

Given the true power, reactive power and apparent power values for the circuit below are 119.33W, 119.96VAR and 169.2VA. Determine the power factor values for the circuit.

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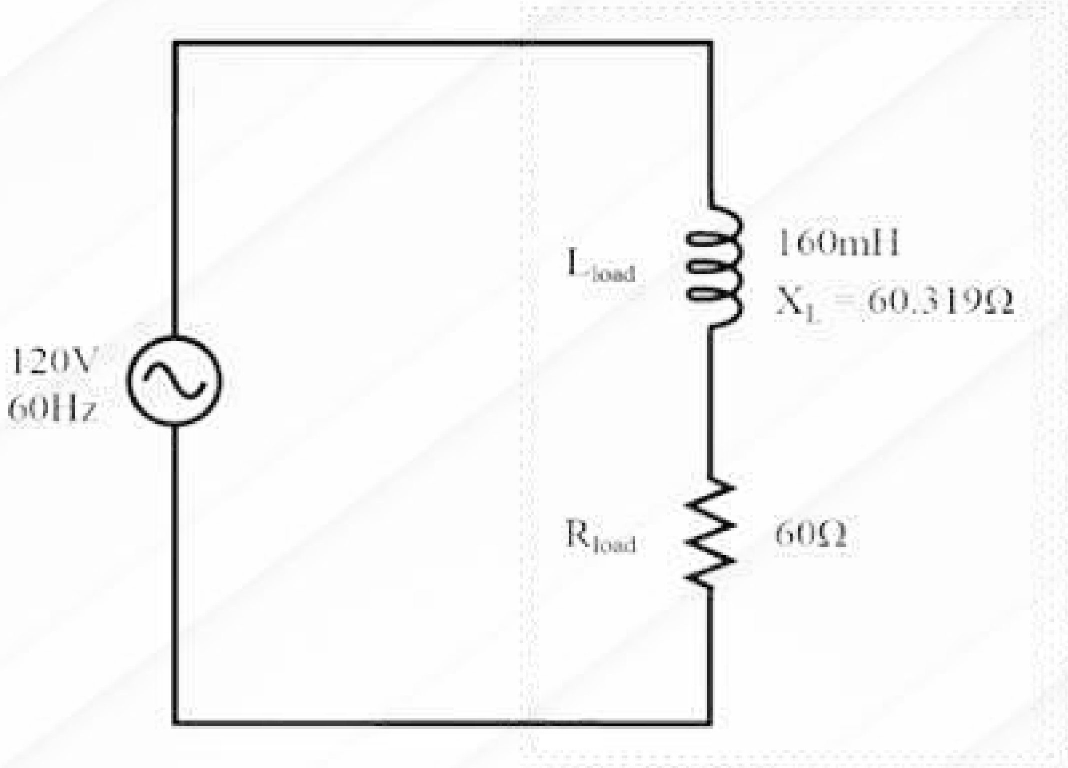

Solution :

$$pf = \frac{Real\ Power}{Apparent\ Power} = \frac{119.33W}{169.2VA} = 0.705$$

Example 6

Calculate the reactive power for the circuit given.

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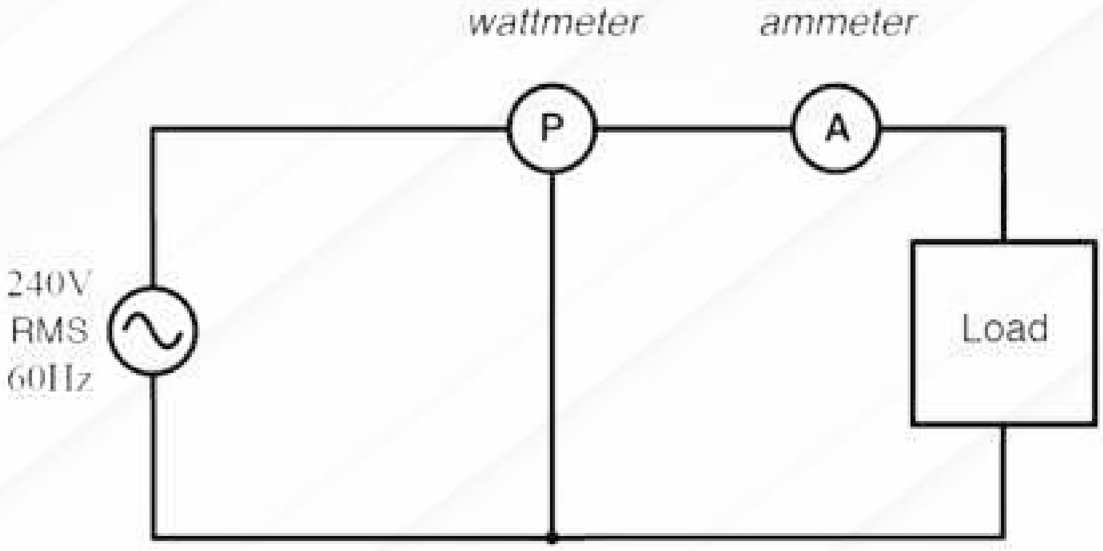

Wattmeter reading = 1.5kW

Ammeter reading (RMS) = 9.615A

Solution :

Apparent Power

$$S = VI = (240)(9.615) = 2307.6VA$$

Reactive Power

$$Q = \sqrt{S^2 - P^2} = \sqrt{2307.6^2 - 1.5k^2} = 1.754kVAR$$

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