Notes of Matrices

ENGINEERING MATHEMATICS 1

DBM10013

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Published Edition 2021

Published by:

Mathematics, Science & Computer Department

Polytechnic Sultan Mizan Zainal Abidin

Dungun, Terengganu Darul Iman

http://www.psmza.edu.my

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PREFACE

Alhamdulillah praise is to Allah S.W.T with His grace and mercy the first edition of an e-book for the topic Matrices in Engineering Mathematics 1 (DBM10013) has finally finished.

With His grace, the e-book based upon the latest curriculum requirement by the Ministry of Education to be used by polytechnic students and lecturers especially.

We wish to thank Politeknik Sultan Mizan Zainal Abidin (PSMZA), Dungun Terengganu for providing a good course for lecturers in other to complete this e-book writing. Infinite thanks to Our Head of Department Mrs. Rosmida Binti Ab Ghani and Head of Mathematics Course Mrs. Rosamalina Binti Mohd @ Mohd Noor who has to give us a chance to write this e-book.

The commitment of group members is always high to be our motivator to prepare this e-book. The members always cooperate, not counting, and are always eager to give ideas for using an e-book. We also like to express our special thanks and appreciation to the lecturers involved in this project. Constructive comments which used to improve future modules will be much appreciated.

Finally, we would also like to thank our colleagues and families for their help and encouragement throughout the preparation of this book. We hope that this e-book can benefit all students and lecturers at the polytechnic and will use as supplementary material for education.

Hasliza Binti Hashim
Wan Eliani Binti Wan Hassan
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ABSTRACT

Matrices e-book is to help students of semester 1 in polytechnics to understand more about this topic contained in the Engineering Mathematics 1 course. Students will be exposed to an example of questions related to matrices that cover sub-topics of Construct Matrices, Demonstrate the Operation of Matrices and Demonstrate Simultaneous Linear equations along with its easy-to-understand solutions. Exercise questions are also provided in this e-book to strengthen students' understanding and skills in achieving the Course Learning Outcome (CLO) set by the Polytechnics. Students need to achieve CLO1, which is they can use a mathematical statement to describe the relationship between various physical phenomena and CLO2 how to show mathematical solutions using the appropriate techniques in mathematics.

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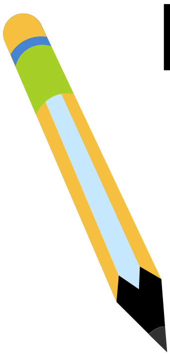
4.0 MATRICES

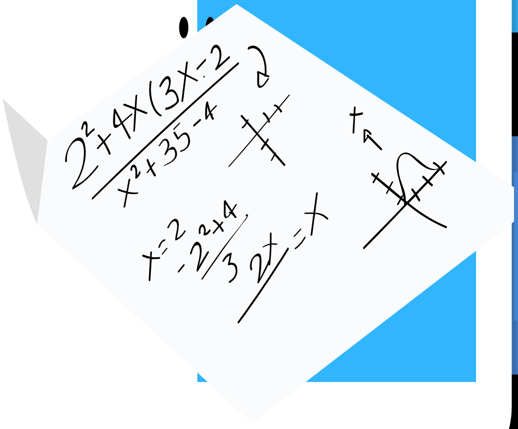
A **matrix** is a **rectangular** array of numbers enclosed in large **brackets.**



For example,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is a **matrix.**

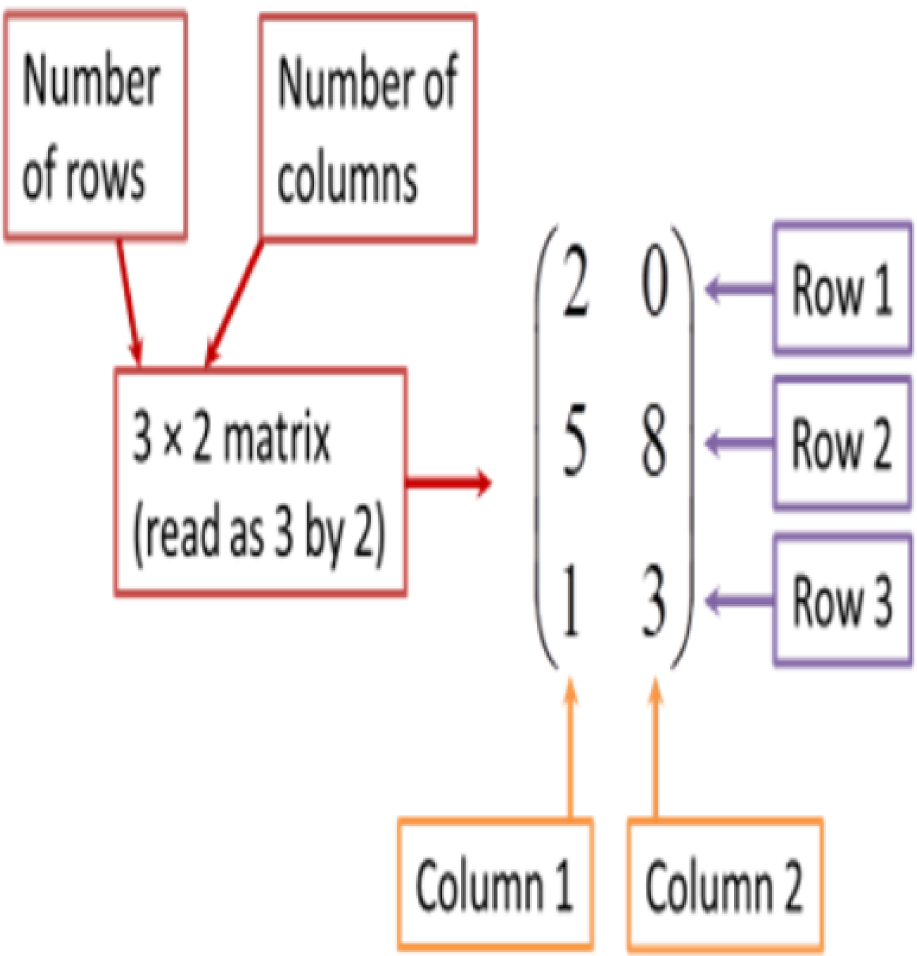


4.1 CONSTRUCT MATRICES

Rows, Columns and Order of Matrices

A matrix which has **m** rows **n** columns

is known as a **matrix of order** $m \times n$.



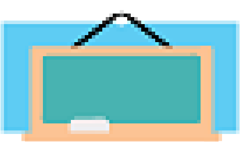
4.1.1 IDENTIFY THE CHARACTER OF MATRICES

a. Elements of a matrix

b. Order of a matrix

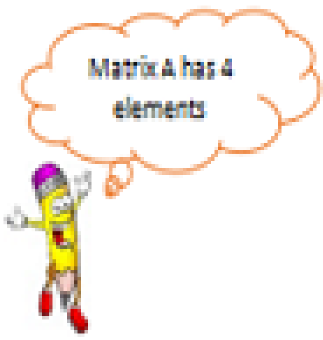
c. Another Types of Matrices

a. Elements of a matrix



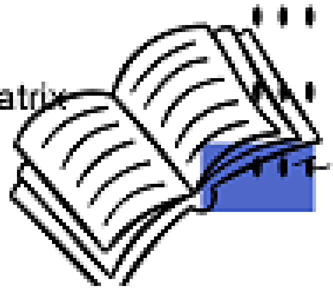
- All the number in a matrix are known as the <u>elements of the matrix.</u>

$$A = \begin{pmatrix} 9 & -4 \\ 0 & 7 \end{pmatrix}$$



- Each element in **i-row** and **j-column** of matrix is labelled as P_{ij}.

P



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b. Order of a matrix

A matrix which has m rows and n columns is called a matrix of order $m \times n$ or $m \times n$ matrix.

For example:

3 x 1 matrix	3 x 3 matrix	1 x 3 matrix
$\begin{pmatrix} 8 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 6 \\ -5 & 4 & 2 \\ 9 & 0 & 4 \end{pmatrix}$	$\begin{pmatrix} 4 & 5 & 6 \end{pmatrix}$

If the matrices above are represented by capital letters A, B and C respectively, then the order of each matrix can be written in subscripts as A_{3x1}, B_{3x3} and C_{1x3}

Example : Given a matrix M = $\begin{pmatrix} 3 & 1 & -1 \\ 6 & 2 & 5 \end{pmatrix}$

State the order of matrix M

M_{2x3}

or

2 x 3 matrix

Find the value of

$m_{21} + m_{13}$

$= 6 + (-1)$

$= 5$

Find the value of

$m_{11} \times m_{22} + m_{13} \times m_{21}$

$= (3 \times 5) + (-1 \times 6)$

$= 15 + (-6)$

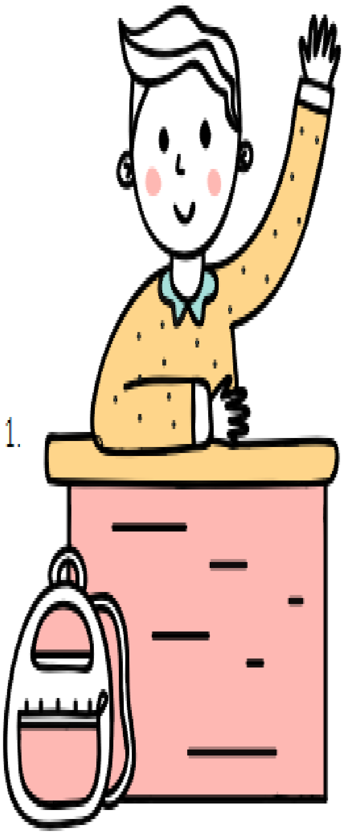
$= 9$

c. Another Types of Matrices

Row Matrix

A matrix that just has a single row, with order $1 \times n$.

Example: $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$



Column Matrix

A matrix that just has a single column, with order $m \times 1$.

Example: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Rectangular Matrix

An array that has a different number of rows and columns, and its order is $m \times n$

Example: $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

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Square Matrix

A square matrix is a matrix where the number of rows is equal to the number of columns. The following examples are square matrices.

2 X 2 3 X 3 4 X 4

$$\begin{pmatrix} 3 & 4 \\ -3 & -5 \end{pmatrix} \quad \begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ -1 & 7 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & -2 & 16 & 9 \\ -4 & 5 & 12 & -23 \\ 0 & 21 & 17 & 3 \\ 1 & -9 & 4 & 15 \end{pmatrix}$$

Zero Matrix

A zero matrix is one which has all its elements zero. Here is a 3x3 zero matrix:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3 \times 3} = \mathbf{0}$$

In mathematics, particularly linear algebra, a **zero matrix** or **null matrix** is a matrix with all its entries being zero. Some examples of zero matrices are

$$0_{1,1} = \begin{bmatrix} 0 \end{bmatrix}, \; 0_{2,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; 0_{2,3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6 1 2 3 4

Diagonal Matrix

If all the elements of a square matrix consist of zeros except the diagonal, then this matrix is called a diagonal matrix. The following examples are diagonal matrices.

$$\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \qquad \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix} \qquad \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Identity Matrix

If all the elements of a diagonal matrix consist of the value 1, then the matrix is an identity matrix. The following examples are identity matrices.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

An identity matrix is special because when you multiply a matrix with it or when you multiply it with a matrix, the matrix does not change. For examples:

$$\boxed{AI = IA = A,\ IB = BI = B}$$

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4.1.2 STATE THE TRANSPOSITION OF A MATRICES

When you interchange the rows of a matrix with its columns, you would have converted a matrix A_{mn} to another matrix A_{nm}. In other words, a matrix of size $m \times n$ will now be of size $n \times m$. This new matrix is called the transpose of a matrix. The symbol for a transpose of a matrix A is A^T. Let's look at the following example.

$$\text{If } A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}, \text{ then } A^T = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}.$$

$$\text{If } A = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & -6 \\ 6 & 0 & -1 \end{pmatrix}, \text{ then } A^T = \begin{pmatrix} 2 & 2 & 6 \\ 0 & 1 & 0 \\ 0 & -6 & -1 \end{pmatrix}$$

The transpose of a transpose is the original matrix, $(A^T)^T = A$

Some important properties relating to transpose are:

$$(AB)^T = B^T A^T$$

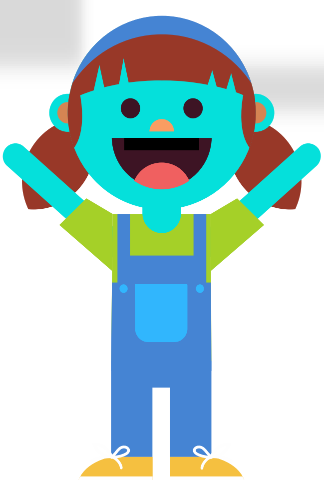
$$(ABC\ldots Z)^T = Z^T \ldots\ldots B^T A^T$$

$$(A + B)^T = A^T + B^T$$

4.2 OPERATION OF MATRICES

4.2 OPERATION OF MATRICES

- 4.2.1 ADDITION OF MATRICES
- 4.2.2 SUBSTRACTION OF MATRICES
- 4.2.3 MULTIPLICATION OF MATRICES
- 4.2.4 DETERMINANT OF MATRICES

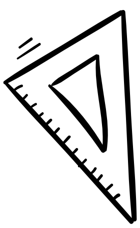


4.2.1 ADDITION OF MATRICES

Addition of matrices can be carried out by adding the corresponding elements of the matrices involved. Only matrices of the same order can be added.

$3+4=7$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$



$-(2)=-2$

$$-\begin{bmatrix} 2 & -4 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ -7 & -10 \end{bmatrix}$$

4.2.2 SUBSTRACTION OF MATRICES

Substraction of matrices can be carried out by substract the corresponding elements of the matrices involved. Only matrices of the same order can be substracted

$3-4=-1$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

4.2.3 MULTIPLICATION OF MATRICES

a. Scalar Multiplication

To multiply a matrix with a real number, we multiply each element with this number

$$2\times\begin{bmatrix}4 & 0\\ 1 & -9\end{bmatrix}=\begin{bmatrix}8 & 0\\ 2 & -18\end{bmatrix}\qquad 2\times4=8$$

Example :

Express $2\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}+3\begin{pmatrix}1 & 2\\ 4 & 5\end{pmatrix}-4\begin{pmatrix}2 & 0\\ 3 & -1\end{pmatrix}$ as a single matrix

Solution:

$$2\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}+3\begin{pmatrix}1 & 2\\ 4 & 5\end{pmatrix}-4\begin{pmatrix}2 & 0\\ 3 & -1\end{pmatrix}$$

Do the scalar multiplication of each matrix first

$$=\begin{pmatrix}4 & 6\\ -2 & 4\end{pmatrix}+\begin{pmatrix}3 & 6\\ 12 & 15\end{pmatrix}-\begin{pmatrix}8 & 0\\ 12 & -4\end{pmatrix}$$

Do the addition and subtraction of matrices from left to right

$$=\begin{pmatrix}7 & 12\\ 10 & 19\end{pmatrix}-\begin{pmatrix}8 & 0\\ 12 & -4\end{pmatrix}$$

$$=\begin{pmatrix}-1 & 12\\ -2 & 23\end{pmatrix}$$

b. Product of Two Matrices

Multiplication of two matrices,

A m_{xn} x B p_{xq}, can only be carried out if n = p ; and the product is a matrix C of order m x q.

$$A_{m \times n} \times B_{p \times q} = C_{m \times q}$$

Must be equal

The illustration below shows how two matrices are multiplied:

Column 1

Sum up all the products of the corresponding elements

Row 1

$$\underset{A_{2\times 2}}{\begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}} \underset{B_{2\times 2}}{\begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}} = \begin{pmatrix} 1(2)+(-1)(1) & 1(0)+(-1)(4) \\ 3(2)+2(1) & 3(0)+2(4) \end{pmatrix} = \underset{C_{2\times 2}}{\begin{pmatrix} 1 & -4 \\ 8 & 8 \end{pmatrix}}$$

STEPS

1. Multiply each element in the row i- row of matrix A by the corresponding element in the j- column of matrix B

2. Sum up all the products obtained in 1. This produces the elements Cij of matrix C.

3. Repeat steps 1 and 2 until the elements from all the rows of matrix A are multiplied by the corresponding elements from all the columns matrix B.

If the number of elements in row vector is **NOT** the same as the number of rows in the second matrix then their product is not defined.

$$\underbrace{\begin{bmatrix}1 & 2 & 3\end{bmatrix}}_{1 \times 3} \cdot \underbrace{\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}}_{2 \times 3} = \textit{NOT DEFINED}$$

TIPS :
$A^2 = A \times A$
Is not the square of each element in matrix

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EXAMPLE

Find the product of each of the following :

a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}\begin{pmatrix} -1 & 4 \end{pmatrix}$

Solution

a) $$\begin{pmatrix} 3 \\ 2 \end{pmatrix}\begin{pmatrix} -1 & 4 \end{pmatrix} = \begin{pmatrix} 3(-1) & 3(4) \\ 2(-1) & 2(4) \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 12 \\ -2 & 8 \end{pmatrix}$$

b) $\begin{pmatrix} -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$

Solution

b) $$\begin{pmatrix} -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -1(2)+3(-3) & -1(1)+3(2) \end{pmatrix}$$

$$= \begin{pmatrix} -11 & 5 \end{pmatrix}$$

c. Matrix equations involving multiplication of two matrices

1. Given $\begin{pmatrix} p & 3 \\ 2 & q \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, find the values of p and q.

Solution

$$\begin{pmatrix} p & 3 \\ 2 & q \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \end{pmatrix},$$

$$\begin{pmatrix} 3p-6 \\ 6-2q \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$3p - 6 = 3$ and $6 - 2q = 2$

$3p = 9$ $\quad -2q = -4$

$p = 3$ $\quad q = 2$

2. Find the values of x and y which satisfy the matrix equation

$$\begin{pmatrix} 2 & x \\ 3 & 0 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix}=\begin{pmatrix} y & 0 \\ -3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} -2+4x & 6+2x \\ -3 & 9 \end{pmatrix}=\begin{pmatrix} y & 0 \\ -3 & 9 \end{pmatrix}$$

$$Thus, 6+2x=0 \qquad And, -2+4x=y$$

$$2x=-6 \qquad -2+4(-3)=y$$

$$x=-3 \qquad y=-14$$

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Activities !!!!

1. Express the following matrices as a single matrix

a) $\begin{pmatrix} -5 & 9 \\ -6 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 2 & -8 \end{pmatrix}$

b) $\begin{pmatrix} 3 \\ -9 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix}$

2. Based on the following matrices,

A = $\begin{pmatrix} 3 & 7 \\ 9 & 5 \end{pmatrix}$ **B** = $\begin{pmatrix} 3 & 4 \\ -3 & -5 \end{pmatrix}$ **C** = $\begin{pmatrix} 4 & 5 \\ 2 & -1 \end{pmatrix}$ **D** = $\begin{pmatrix} 4 & -2 \\ 5 & 7 \end{pmatrix}$

Determine:

a. **A** + **B**

b. **A** – **C**

c. **D** + (**B** – **A**)

d. **B** + **C**

3. Given that **A** = $\begin{pmatrix} -2 & -1 \\ 4 & 1 \end{pmatrix}$ and **B** = $\begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$, Find **AB** and **BA**.

4. Find the values of x and y for the followings:

a) $\begin{pmatrix} x & 2 \\ 1 & y \end{pmatrix} + \begin{pmatrix} 2x & 3 \\ 2 & y \end{pmatrix} = \begin{pmatrix} 9 & 5 \\ 3 & 8 \end{pmatrix}$

b) $\begin{pmatrix} x & 1 & 5 \\ 0 & 1 & 7 \\ 0 & y & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 1 & 5 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 6 & 2 \\ 1 & 5 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

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4.2.4 DETERMINANT OF MATRICES

The determinant of a square matrix is a special number that can be calculated from the matrix. It is used to represent the real-value of the matrix which can be used to solve simple algebra problems later on. The symbol for the determinant of matrix A is det(A) or |A|.

a. Determinant of a 2 X 2 matrix

For a matrix of size **2 x 2**, the method to find the determinant is:

If $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

then, $\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$.

b. Determinant of a 3 X 3 matrix

The determinant of a 3x3 matrix is a little more tricky and is found as follows (for this case assume A is an arbitrary 3x3 matrix A, where the elements are given below)

If $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$,

then $|\mathbf{A}| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{12} & a_{23} \\ a_{13} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

therefore, $|\mathbf{A}| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

EXAMPLE

1

If $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, determine det(**A**).

Solution

$$|A| = \begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = (5(8) - 6(7))$$
$$= (40 - 42)$$
$$= -2$$

2

Determine the determinant of matrix $\begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$

Solution

$$|A| = 1\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$
$$= 1(3(2) - 0(1)) - 3(4(2) - 0(2)) + 2(4(1) - 3(2))$$
$$= 1(6 - 0) - 3(8 - 0) + 2(4 - 6)$$
$$= 1(6) - 3(8) + 2(-2)$$
$$= -22$$

Activities !!!!

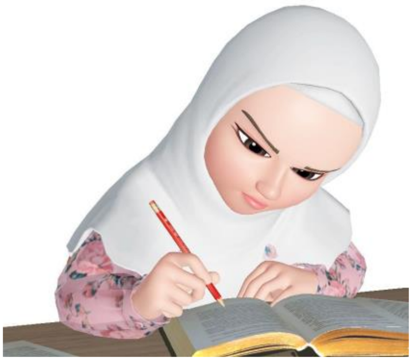
1. Determine the determinants for the following 2x2 matrices:

a) $\begin{pmatrix} 6 & 13 \\ 4 & 12 \end{pmatrix}$ b) $\begin{pmatrix} -3 & 8 \\ 5 & 3 \end{pmatrix}$ c) $\begin{pmatrix} 4 & 2 \\ -6 & 3 \end{pmatrix}$

2. Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 5 & 3 \\ 1 & 0 & 4 \\ 7 & -2 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$

Determine:

a) $|\mathbf{A}|$ b) $|\mathbf{B}|$ c) $|\mathbf{C}|$



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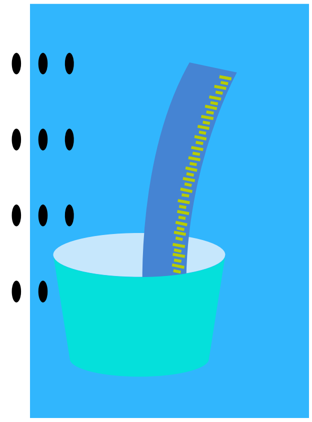
4.3 DEMONSTRATE SIMULTANEOUS LINEAR EQUATIONS

SIMULTANEOUS LINEAR EQUATIONS

- INVERSE METHOD
- CRAMER'S RULE METHOD



4.3.1 INVERSE METHOD

STEPS

1. **Write in a matrix form Ax = b**
2. Find the determinant of A
3. Find the minor of A
4. Find the of cofactor of A
5. Find the of adjoin of A
6. Find the of inverse of A, A^{-1}
7. Find the of THREE (3) variables by using the $x=A^{-1}b$

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EXAMPLE

Solve the simultaneous linear equations systems below:

$$\begin{aligned} x + 3y + 3z &= 4 \\ 2x - 3y - 2z &= 2 \\ 3x + y + 2z &= 5 \end{aligned}$$

Step 1: Write in a matrix form Ax = b

$$\underbrace{\begin{pmatrix} 1 & 3 & 3 \\ 2 & -3 & -2 \\ 3 & 1 & 2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}}_{b}$$

Step 2: Find the Determinant of A

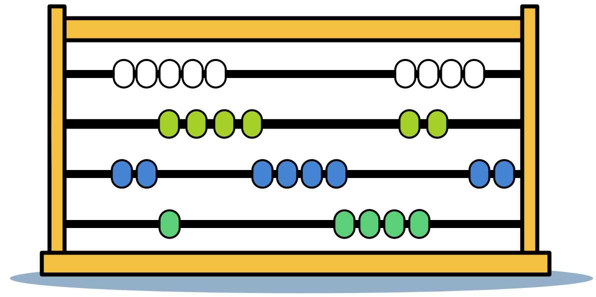
$$A = \begin{pmatrix} 1 & 3 & 3 \\ 2 & -3 & -2 \\ 3 & 1 & 2 \end{pmatrix}$$

$$|A| = 1\begin{vmatrix} -3 & -2 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= -1$$

1 2 3 4

Step 3: Find the Minor of A .



$$A = \begin{pmatrix} 1 & 3 & 3 \\ 2 & -3 & -2 \\ 3 & 1 & 2 \end{pmatrix}$$

$$M_{11} = \begin{vmatrix} -3 & -2 \\ 1 & 2 \end{vmatrix} = -6-(-2) = -4$$ by removing row 1 and column 1 from **A**

$$M_{12} = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 4-(-6) = 10$$ by removing row 1 and column 2 from **A**

$$M_{13} = \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} = 2-(-9) = 11$$ by removing row 1 and column 3 from **A**

$$M_{21} = \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} = 6-3 = 3$$ by removing row 2 and column 1 from **A**

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = 2-9 = -7$$ by removing row 2 and column 2 from **A**

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1-9 = -8$$ by removing row 2 and column 3 from **A**

$$M_{31} = \begin{vmatrix} 3 & 3 \\ -3 & -2 \end{vmatrix} = -6-(-9) = 3$$ by removing row 3 and column 1 from **A**

$$M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = -2-6 = -8$$ by removing row 3 and column 2 from **A**

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = -3-6 = -9$$ by removing row 3 and column 3 from **A**

$$\text{Minor } \mathbf{A} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$

So, $$\text{Minor } \mathbf{A} = \begin{pmatrix} -4 & 10 & 11 \\ 3 & -7 & -8 \\ 3 & -8 & -9 \end{pmatrix}$$

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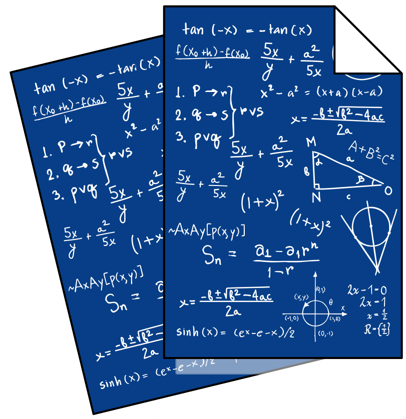

Step 4: Find the Cofactor of A

$$\text{Cofactor } \mathbf{A} = \begin{pmatrix} (-1)^2 M_{11} & (-1)^3 M_{12} & (-1)^4 M_{13} \\ (-1)^3 M_{21} & (-1)^4 M_{22} & (-1)^5 M_{23} \\ (-1)^4 M_{31} & (-1)^5 M_{32} & (-1)^6 M_{33} \end{pmatrix}$$

$$\text{Cofactor } \mathbf{A} = \begin{pmatrix} (-1)^2 -4 & (-1)^3 10 & (-1)^4 11 \\ (-1)^3 3 & (-1)^4 -7 & (-1)^5 -8 \\ (-1)^4 3 & (-1)^5 -8 & (-1)^6 -9 \end{pmatrix}$$

$$\text{So, Cofactor } \mathbf{A} = \begin{pmatrix} -4 & -10 & 11 \\ -3 & -7 & 8 \\ 3 & 8 & -9 \end{pmatrix}$$

This 4 elements will be change the value of +ve and –ve from the Minor **A.**

Step 5: Find the Adjoint of A

$$\text{Adjoint } \mathbf{A} = \begin{pmatrix} -4 & -3 & 3 \\ -10 & -7 & 8 \\ 11 & 8 & -9 \end{pmatrix}$$

Transpose the Cofactor **A** to get the Adjoint **A.**

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Step 6: Find the Inverse of A, A^{-1}.

Using the formula below:

$$A^{-1} = \frac{1}{|A|} \text{Adjoint } A$$

$$A^{-1} = \frac{1}{|-1|}\begin{pmatrix} -4 & -3 & 3 \\ -10 & -7 & 8 \\ 11 & 8 & -9 \end{pmatrix}$$

So, Inverse A, $A^{-1} = \begin{pmatrix} 4 & 3 & -3 \\ 10 & 7 & -8 \\ -11 & -8 & 9 \end{pmatrix}$

Step 7: Find the value of x,y and z by using $x = A^{-1}b$.

$$x = A^{-1}b.$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & 3 & -3 \\ 10 & 7 & -8 \\ -11 & -8 & 9 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 14 \\ -15 \end{pmatrix}$$

$\therefore x = 7, y = 14, z = -15$

4.3.2 CRAMER'S RULE METHOD

Steps:

1. Write in a matrix form Ax = b
2. Calculate the determinant of A ,$|A|$
3. Find A_1 by substituting 'b' into column 1 of A. Calculate $|A_1|$
4. Find A_2 by substituting 'b' into column 2 of A. Calculate $|A_2|$
5. Find A_3 by substituting 'b' into column 3 of A. Calculate $|A_3|$
6. Find the of THREE (3) variables by using the formula :

$$x=\frac{|A_1|}{|A|}, \; y=\frac{|A_2|}{|A|}, \; z=\frac{|A_3|}{|A|}$$

Consider the systems of simultaneous linear equations below:

$$a_{11}x + a_{12}y + a_{13}z = b_1$$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

Write it in a matrix form, **Ax = b**.

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}_{\mathbf{b}}$$

$$\mathrm{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

You get A_1, by substituting $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ into column 1 of matrix A.

Therefore $A_1 = \begin{pmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{pmatrix}$

Using the same method, therefore $A_2 = \begin{pmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{pmatrix}$ and $A_3 = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{pmatrix}$

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EXAMPLE

Solve the simultaneous linear equations systems below:

$$5x - y + 7z = 4$$
$$6x - 2y + 9z = 5$$
$$2x + 8y - 4z = 8$$

Step 1: Write in a matrix form Ax = b

$$\underbrace{\begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}}_{b}$$

Step 2: Find the Determinant of A

$$A = \begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}.$$

$$|A| = 5\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 9 \\ 2 & -4 \end{vmatrix} + 7\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$
$$= 2$$

Step 3: Find A_1 by substituting 'b' into column 1 of A. Calculate $|A_1|$

$$A_1 = \begin{pmatrix} 4 & -1 & 7 \\ 5 & -2 & 9 \\ 8 & 8 & -4 \end{pmatrix}$$

$$|A_1| = 4\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} - (-1)\begin{vmatrix} 5 & 9 \\ 8 & -4 \end{vmatrix} + 7\begin{vmatrix} 5 & -2 \\ 8 & 8 \end{vmatrix}$$
$$= 44$$

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Step 4: Find A_2 by substituting 'b' into column 2 of A. Calculate $|A_2|$

$$A_2 = \begin{pmatrix} 5 & 4 & 7 \\ 6 & 5 & 9 \\ 2 & 8 & -4 \end{pmatrix}$$

$$|A_2| = 5\begin{vmatrix} 5 & 9 \\ 8 & -4 \end{vmatrix} - 4\begin{vmatrix} 6 & 9 \\ 2 & -4 \end{vmatrix} + 7\begin{vmatrix} 6 & 5 \\ 2 & 8 \end{vmatrix}$$
$$= -26$$

Step 5: Find A_3 by substituting 'b' into column 3 of A. Calculate $|A_3|$

$$A_3 = \begin{pmatrix} 5 & -1 & 4 \\ 6 & -2 & 5 \\ 2 & 8 & 8 \end{pmatrix}$$

$$|A_3| = 5\begin{vmatrix} -2 & 5 \\ 8 & 8 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 5 \\ 2 & 8 \end{vmatrix} + 4\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$
$$= -34$$

Step 6: Find the 3 variables using the formula

$$x = \frac{|A_1|}{|A|} = \frac{44}{2} = 22$$

$$y = \frac{|A_2|}{|A|} = \frac{-26}{2} = -13$$

$$z = \frac{|A_3|}{|A|} = \frac{-34}{2} = -17$$

$\therefore x = 22, y = -13, z = -17$

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Activities !!!!

1. Solve the following system of linear equations using the inverse method

$$\begin{aligned} 2x + y + z &= 8 \\ 5x - 3y + 2z &= 3 \\ 7x + y + 3z &= 20 \end{aligned}$$

$$\begin{aligned} 3x + 2y + 4z &= 3 \\ x + y + z &= 2 \\ 2x - y + 3z &= -3 \end{aligned}$$

2. Solve the following system of linear equations using the Cramer's Rule.

$$\begin{aligned} x + 2y - z &= 4 \\ 3x - 4y - 2z &= 2 \\ 5x + 3y + 5z &= -1 \end{aligned}$$

$$\begin{aligned} 4a - 5b + 6c &= 3 \\ 8a - 7b - 3c &= 9 \\ 7a - 8b + 9c &= 6 \end{aligned}$$

3. Solve the following system of linear equations using the inverse method and Cramer's Rule.

$$\begin{aligned} x + y + 2z &= 1 \\ 2x + 3y + 6z &= 1 \\ 3x + 2y - 4z &= 2 \end{aligned}$$

$$\begin{aligned} 3x + 2y - z &= 10 \\ 7x - y + 6z &= 8 \\ 3x + 2z - 5 &= 0 \end{aligned}$$

EXERCISES

1. Determine the order of the following matrices:

$$\begin{pmatrix} 4 \\ -6 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} 12 & -9 \\ 6 & 10 \end{pmatrix}$$

2. Given that $E = \begin{pmatrix} 4 \\ -6 \\ 5 \end{pmatrix}$ $F = \begin{pmatrix} 20 \\ -5 \\ 6 \end{pmatrix}$. Find $E + F$

3. If $A = \begin{pmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{pmatrix}$. Find AB.

4. Given that $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -1 & 3 \\ -1 & 2 \end{pmatrix}$

5. Given $A = \begin{pmatrix} -1 & 3 & 3 \\ 2 & -2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & -1 & 3 \\ 2 & 0 & 4 \\ 2 & 5 & 0 \end{pmatrix}$

Calculate: a) Determinant $|B|$

b) $B^T - A^T$

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e ISBN 978-967-2099-65-9